UNIT 3
Polynomials and Nonlinear Functions

Focus
Use quadratic and other nonlinear functions to represent and model problem situations and to analyze and interpret relationships.

CHAPTER 7 Polynomials
BIG Idea Understand there are situations modeled by functions that are not linear, and model the situations.

CHAPTER 8 Factoring
BIG Idea Use algebraic skills to simplify algebraic expressions, and solve equations and inequalities in problem situations.

CHAPTER 9 Quadratic and Exponential Functions
BIG Idea Understand there is more than one way to solve a quadratic equation and solve them using appropriate methods.
BIG Idea Understand there are situations modeled by functions that are neither linear nor quadratic, and model the situations.
Cross-Curricular Project

Algebra and Physical Science

Out of this World  You can probably name the planets in the solar system, but can you name planets outside of our system? In recent years, planets in other systems have been discovered. In August, 2004, a team of astronomers discovered a small planet orbiting a star known as 55 Cancri. Star 55 Cancri has three other planets, making it the first known four-planet system outside our system. In this project, you will examine how exponents, factors, and graphs are useful in presenting information about planets.

Math Online Log on to algebra1.com to begin.
Chapter 7
Polynomials

Real-World Link
Running Polynomials can be used to model many real-world situations, such as the way that distance runners on a curved track should be staggered at the start of a race.

Key Vocabulary
- binomial (p. 376)
- FOIL method (p. 399)
- monomial (p. 358)
- polynomial (p. 376)

Foldables
Study Organizer
Polynomials Make this Foldable to help you organize information about polynomials. Begin with a sheet of 11” by 17” paper.

1. Fold in thirds lengthwise.
2. Open and fold a 2” tab along the width. Then fold the rest in fourths.
3. Draw lines along folds and label as shown.

Big Ideas
- Find products and quotients of monomials.
- Find the degree of a polynomial and arrange the terms in order.
- Add, subtract, and multiply polynomial expressions.
- Find special products of binomials.
GET READY for Chapter 7

Diagnose Readiness  You have two options for checking Prerequisite Skills.

Option 2

Take the Online Readiness Quiz at algebra1.com.

Option 1

Take the Quick Check below. Refer to the Quick Review for help.

**Write each expression using exponents.**  (Lesson 1-1)

1. 2 · 2 · 2 · 2
2. 3 · 3 · 3
3. 5 · 5
4. x · x · x
5. a · a · a · a · a
6. (x · y · y · y
7. \( \frac{1}{2} \) · \( \frac{1}{2} \) · \( \frac{1}{2} \) · \( \frac{1}{2} \)
8. \( \frac{a}{b} \) · \( \frac{a}{b} \) · \( \frac{c}{d} \) · \( \frac{c}{d} \) · \( \frac{c}{d} \)

**Evaluate each expression.**  (Lesson 1-1)

9. 3²
10. 4³
11. (−6)²
12. (−3)³
13. \( \left( \frac{2}{3} \right)^4 \)
14. \( \left( \frac{-7}{8} \right)^2 \)

15. **PROBABILITY**  The probability of correctly guessing the outcome of a flipped penny six times is \( \left( \frac{1}{2} \right)^6 \). Express this probability as a fraction without exponents.

**Find the area or volume of each figure.**  (Prerequisite Skill)

16.
17. 4 ft
18.
19. 2 ft

**EXAMPLE 1**

Express 6 · 6 · 6 · x · x + y · y · y · y · z using exponents.

3 factors of six is 6³. 4 factors of y is y⁴. 2 factors of x is x². 1 factor of z is z or z.

So, 6 · 6 · 6 · x · x + y · y · y · y · z = 6³x² + y⁴z.

**EXAMPLE 2**

Evaluate \( \left( \frac{8}{11} \right)^2 \).

\[ \left( \frac{8}{11} \right)^2 = \frac{64}{121} \]  Simplify.

**EXAMPLE 3**

Find the volume of the figure.

\[ V = \ell \cdot w \cdot h \]

Substitute 3 for length, 4 for width, and 2 for height.

\[ = 3 \cdot 4 \cdot 2 \]

Evaluate volume.

The volume of the box is 24 cubic feet.
The table shows the braking distance for a vehicle at certain speeds. If \( s \) represents the speed in miles per hour, then the approximate number of feet that the driver must apply the brakes is \( \frac{1}{20} s^2 \). Notice that when speed is doubled, the braking distance is quadrupled.

**Multiply Monomials** A monomial is a number, a variable, or a product of a number and one or more variables like \( \frac{1}{20}s^2 \). An expression like \( \frac{x}{2y} \), which involves the division of variables is not a monomial. Monomials that are real numbers are called constants.

**EXAMPLE** Identify Monomials

Determine whether each expression is a monomial. Explain your reasoning.

<table>
<thead>
<tr>
<th>Expression</th>
<th>Monomial?</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-5)</td>
<td>yes</td>
<td>(-5) is a real number and an example of a constant.</td>
</tr>
<tr>
<td>(p + q)</td>
<td>no</td>
<td>The expression involves the addition, not the product, of two variables.</td>
</tr>
<tr>
<td>(x)</td>
<td>yes</td>
<td>Single variables are monomials.</td>
</tr>
</tbody>
</table>

Recall that an expression of the form \( x^n \) is called a power and represents the product you obtain when \( x \) is used as a factor \( n \) times. The word power is also used to refer to the exponent itself. The number \( x \) is the base, and the number \( n \) is the exponent.
In the following examples, the definition of a power is used to find the products of powers. Look for a pattern in the exponents.

\[
\begin{align*}
2^3 \cdot 2^5 &= 2^8 \\
3^2 \cdot 3^4 &= 3^6
\end{align*}
\]

These examples suggest the property for multiplying powers.

**KEY CONCEPT**

**Product of Powers**

**Words** To multiply two powers that have the same base, add their exponents.

**Symbols** For any number \(a\) and all integers \(m\) and \(n\), \(a^m \cdot a^n = a^{m+n}\).

**Example** \(a^4 \cdot a^{12} = a^{4+12} = a^{16}\)

**EXAMPLE**

**Product of Powers**

Simplify each expression.

**a.** \((5x^7)(x^6)\)

\[
(5x^7)(x^6) = (5)(1)(x^7)(x^6) = 5x^{13}
\]

**b.** \((4ab^6)(-7a^2b^3)\)

\[
(4ab^6)(-7a^2b^3) = (4)(-7)(a \cdot a^2)(b^6 \cdot b^3) = -28a^{1+2}b^{6+3} = -28a^3b^9
\]

**Powers of Monomials** You can also look for a pattern to discover the property for finding the power of a power.

\[
\begin{align*}
(4^2)^5 &= 4^{2 \cdot 5} = 4^{10} \\
(z^8)^3 &= z^{8 \cdot 3} = z^{24}
\end{align*}
\]

These examples suggest the property for finding the power of a power.
EXAMPLE

**Power of a Power**

Simplify \((3^2)^3\)^2.

\[
(3^2)^3 = (3^2)^6 \\
= 3^{12} \text{ or } 531,441
\]

3. Simplify \((2^2)^4\).

Look for a pattern in these examples.

\[
(xy)^4 = (x \cdot y \cdot x \cdot y)(x \cdot y \cdot x \cdot y) = x^4y^4
\]

\[
(6ab)^3 = (6ab)(6ab)(6ab) = 6^3a^3b^3 \text{ or } 216a^3b^3
\]

These examples suggest the following property.

**KEY CONCEPT**

**Power of a Product**

**Words** To find the power of a product, find the power of each factor and multiply.

**Symbols** For all numbers \(a\) and \(b\) and any integer \(m\), \((ab)^m = a^mb^m\).

**Example** \((-2xy)^3 = (-2)^3x^3y^3 \text{ or } -8x^3y^3\)

EXAMPLE

**Power of a Product**

**GEOMETRY** Express the area of the square as a monomial.

Area \(= s^2\) Formula for the area of a square

\(= (4ab)^2\) Replace \(s\) with \(4ab\).

\(= 4^2a^2b^2\) Power of a Product

\(= 16a^2b^2\) Simplify.

The area of the square is \(16a^2b^2\) square units.

4. Express the area of a square with sides of length \(2xy^2\) as a monomial.

**CONCEPT SUMMARY**

**Simplifying Expressions**

To simplify an expression involving monomials, write an equivalent expression in which:

- each base appears exactly once,
- there are no powers of powers, and
- all fractions are in simplest form.
Lesson 7-1 Multiplying Monomials

**Example 5 (p. 361)**

Simplify.

5. Simplify \( \left( \frac{1}{2}a^2b^2 \right)^3 \) \([-4b]^2\).

**Check Your Understanding**

**Example 1 (p. 358)**

Determine whether each expression is a monomial. Write yes or no. Explain.

1. \( 5 - 7d \)
2. \( \frac{4a}{3b} \)
3. \( n \)

**Examples 2, 3 (pp. 359–360)**

Simplify.

4. \( x(x^4)(x^6) \)
5. \( (4a^4b)(9a^2b^3) \)
6. \( [2^3]^2 \)
7. \( [(3^2)^2]^2 \)
8. \( (3y^5z)^2 \)
9. \( (-2f^2g)^3 \)

**Example 4 (p. 360)**

**GEOMETRY** Express the area of each triangle as a monomial.

10.

11.

**Example 5 (p. 361)**

Simplify.

12. \( (-2v^3w^4)^3(-3vw^3)^2 \)
13. \( (5x^2y)^2 (2xy^3z)^3 (4xyz) \)

**Exercises**

Determine whether each expression is a monomial. Write yes or no. Explain.

14. 12
15. \( 4x^3 \)
16. \( a - 2b \)
17. \( 4n + 5m \)
18. \( \frac{x}{y^2} \)
19. \( \frac{1}{5}abc^{14} \)

Simplify.

20. \( (ab^4)(ab^2) \)
21. \( (p^5q^4)(p^2q) \)
22. \( (-7c^3d^4)(4cd^3) \)
23. \( (-3j^7k^5)(-8jk^8) \)
24. \( (9pq)^2 \)
25. \( (7b^3c^6)^3 \)
26. \( [(3^2)^4]^2 \)
27. \( [(4^3)^3]^2 \)
28. \( [(-2xy^2)^3]^2 \)

**GEOMETRY** Express the area of each figure as a monomial.

29.

30.
Simplify.

31. \((4cd)^2(-3d^2)^3\)

32. \((-2x^5)^2(-5xy^6)^2\)

33. \((2ag^2)^4(3a^2g^3)^2\)

34. \((2m^2n^3)^3(3m^3n)^4\)

35. Simplify the expression \((-2b^3)^4 - 3(-2b^4)^3\).

36. Simplify the expression \(2(-5y^3)^2 + (-3y^3)^3\).

37. **CHEMISTRY** Lemon juice is 10^2 times as acidic as tomato juice. Tomato juice is 10^3 times as acidic as egg whites. How many times as acidic is lemon juice as egg whites? Write as a monomial.

38. **GEOLOGY** The seismic waves of a magnitude 6 earthquake are 10^2 times as great as a magnitude 4 earthquake. The seismic waves of a magnitude 4 earthquake are 10 times as great as a magnitude 3 earthquake. How many times as great are the seismic waves of a magnitude 6 earthquake as those of a magnitude 3 earthquake? Write as a monomial.

Simplify.

39. \((5a^2b^3c^4)(6a^3b^4c^2)\)

40. \((10xy^5z^3)(3x^4y^6z^3)\)

41. \((0.5x^3)^2\)

42. \((0.4h^5)^3\)

43. \((-\frac{3}{4}c)^3\)

44. \((\frac{4}{5}a^2)^2\)

45. \((8y^3)(-3x^2y^2)(\frac{3}{8}xy^4)\)

46. \((\frac{4}{7}m)^2(49m)(17p)(\frac{1}{34}p^5)\)

**GEOMETRY** Express the volume of each solid as a monomial.

47. \(4k^3 \quad 4k^3\)

48. \(x^2y \quad y\)

49. \(2n \quad 4n^3\)

**TELEPHONES** For Exercises 50 and 51, use the following information.
The first transatlantic telephone cable has 51 amplifiers along its length. Each amplifier strengthens the signal on the cable 10^6 times.

50. After it passes through the second amplifier, the signal has been boosted 10^6 \cdot 10^6 times. Simplify this expression.

51. Represent the number of times the signal has been boosted after it has passed through the first four amplifiers as a power of 10^6. Then simplify the expression.

**DEMOLITION DERBY** For Exercises 52 and 53, use the following information.
When a car hits an object, the damage is measured by the collision impact. For a certain car, the collision impact \(I\) is given by \(I = 2s^2\), where \(s\) represents the speed in kilometers per minute.

52. What is the collision impact if the speed of the car is 1 kilometer per minute? 2 kilometers per minute? 4 kilometers per minute?

53. As the speed doubles, explain what happens to the collision impact.
TESTING  For Exercises 54 and 55, use the following information.
A history test covers two chapters. There are $2^{12}$ ways to answer the 12 true-false questions on the first chapter and $2^{10}$ ways to answer the 10 true-false questions on the second chapter.

54. How many ways are there to answer all 22 questions on the test?

55. If a student guesses on each question, what is the probability of answering all questions correctly?

56. OPEN ENDED Write three different expressions that are equivalent to $x^6$.

CHALLENGE Determine whether each statement is true or false. If true, explain your reasoning. If false, give a counterexample.

57. For any real number $a$, $(-a)^2 = -a^2$.

58. For all real numbers $a$ and $b$, and all integers $m$, $n$, and $p$, $(a^m b^n)^p = a^{mp} b^{np}$.

59. For all real numbers $a$, $b$, and all integers $n$, $(a + b)^n = a^n + b^n$.

60. FIND THE ERROR Nathan and Poloma are simplifying $(5^2)(5^9)$. Who is correct? Explain your reasoning.

61. REASONING Compare each pair of monomials. Explain why each pair is or is not equivalent.
   a. $5m^2$ and $(5m)^2$
   b. $(yz)^4$ and $y^4 z^4$
   c. $-3a^2$ and $(-3a)^2$
   d. $2(c^7)^3$ and $8c^{21}$

62. Writing in Math Use the data about braking distances on page 358 to explain why doubling speed quadruples braking distance.

63. The length of a rectangle is three times the width of the rectangle. If the width of the rectangle is $y$ units, what is the area of the rectangle?
   A. $3y$ units$^2$
   B. $3y^2$ units$^2$
   C. $y + 3$ units$^2$
   D. $3y(y + 3)$ units$^2$

64. REVIEW △ABC has coordinates $A(4, 5)$, $B(1, 3)$, and $C(4, 0)$. What will the coordinates of $A'$ be if the triangle is translated 3 units down and 2 units to the left?
   F. $(1, 3)$
   G. $(2, 2)$
   H. $(7, 7)$
   J. $(8, 0)$
Solve each system of inequalities by graphing. (Lesson 6-8)

65. \( y \leq 2x + 2 \)  
   \( y \geq -x - 1 \)

66. \( y \geq x - 2 \)  
   \( y < 2x - 1 \)

67. \( x > -2 \)  
   \( y < x + 3 \)

Determine which ordered pairs are part of the solution set for each inequality. (Lesson 6-7)

68. \( y \leq 2x \), \{(1, 4), (-1, 5), (5, -6), (-7, 0)\}

69. \( y < 8 - 3x \), \{(-4, 2), (-3, 0), (1, 4), (1, 8)\}

Solve each compound inequality. Then graph the solution set. (Lesson 6-4)

70. \( 4 + h \leq -3 \) or \( 4 + h \geq 5 \)

71. \( 4 < 4a + 12 < 24 \)

72. \( 14 < 3h + 2 < 2 \)

73. \( 2m - 3 > 7 \) or \( 2m + 7 > 9 \)

Use elimination to solve each system of equations. (Lesson 5-4)

74. \(-4x + 5y = 2\)  
   \(x + 2y = 6\)

75. \(3x + 4y = -25\)  
   \(2x - 3y = 6\)

76. \(x + y = 20\)  
   \(4 = 0.4x + 0.15y\)

Write an equation in function notation for each relation. (Lesson 3-5)

77. 

78. 

Express the relation shown in each table, mapping, or graph as a set of ordered pairs. Then write the inverse of the relation. (Lesson 3-1)

82. TRANSPORTATION Two trains leave York at the same time, one traveling north, the other south. The northbound train travels at 40 miles per hour and the southbound at 30 miles per hour. In how many hours will the trains be 245 miles apart? (Lesson 2-9)

PREREQUISITE SKILL Simplify. (Pages 694–695)

83. \( \frac{2}{6} \)

84. \( \frac{3}{15} \)

85. \( \frac{10}{5} \)

86. \( \frac{27}{9} \)

87. \( \frac{14}{36} \)

88. \( \frac{9}{48} \)

89. \( \frac{44}{32} \)

90. \( \frac{45}{18} \)
Algebra Lab
Investigating Surface Area and Volume

ACTIVITY

• Cut out the pattern shown from a sheet of centimeter grid paper. Fold along the dashed lines and tape the edges together to form a rectangular prism.

• Find the surface area $SA$ of the prism by counting the squares on all the faces of the prism or by using the formula $SA = 2wl + 2wh + 2lh$, where $w$ is the width, $l$ is the length, and $h$ is the height of the prism.

• Find the volume $V$ of the prism by using the formula $V = lwh$.

• Now construct another prism with dimensions that are 2 times each of the dimensions of the first prism, or 4 centimeters by 10 centimeters by 6 centimeters.

• Finally, construct a third prism with dimensions that are 3 times each of the dimensions of the first prism.

ANALYZE THE RESULTS

1. Copy and complete the table using the prisms you made.

<table>
<thead>
<tr>
<th>Prism</th>
<th>Dimensions</th>
<th>Surface Area (cm$^2$)</th>
<th>Volume (cm$^3$)</th>
<th>Surface Area Ratio</th>
<th>Volume Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original</td>
<td>2 by 5 by 3</td>
<td>62</td>
<td>30</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>4 by 10 by 6</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>6 by 15 by 9</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. MAKE A CONJECTURE Suppose you multiply each dimension of a prism by 2. What is the ratio of the surface area of the new prism to the surface area of the original prism? What is the ratio of the volumes?

3. If you multiply each dimension of a prism by 3, what is the ratio of the surface area of the new prism to the surface area of the original? What is the ratio of the volumes?

4. Suppose you multiply each dimension of a prism by $a$. Make a conjecture about the ratios of surface areas and volumes.

5. Repeat the activity using cylinders. To start, make a cylinder with radius 4 centimeters and height 5 centimeters. To compute surface area $SA$ and volume $V$, use the formulas $SA = 2\pi r^2 + 2\pi rh$ and $V = \pi r^2 h$, where $r$ is the radius and $h$ is the height of the cylinder. Do the conjectures you made in Exercise 4 hold true for cylinders? Explain.
Dividing Monomials

Main Ideas
- Simplify expressions involving the quotient of monomials.
- Simplify expressions containing negative exponents.

New Vocabulary
zero exponent
negative exponent

To test whether a solution is a base or an acid, chemists use a pH test. This test measures the concentration $c$ of hydrogen ions (in moles per liter) in the solution.

$$c = \left( \frac{1}{10} \right)^{\text{pH}}$$

The table gives examples of solutions with various pH levels. You can find the quotient of powers and use negative exponents to compare measures on the pH scale.

Quotients of Monomials  Look for a pattern in the examples below.

$$\frac{4^5}{4^3} = \frac{4 \cdot 4 \cdot 4 \cdot 4 \cdot 4}{4 \cdot 4 \cdot 4} = 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4$$

$$\frac{3^6}{3^2} = \frac{3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3}{3 \cdot 3} = 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$$

KEY CONCEPT

Quotient of Powers

Words  To divide two powers with the same base, subtract the exponents.

Symbols  For all integers $m$ and $n$ and any nonzero number $a$, $\frac{a^m}{a^n} = a^{m-n}$.

Example  $\frac{b^{15}}{b^7} = b^{15-7}$ or $b^8$

EXAMPLE  Quotient of Powers

1. Simplify $\frac{a^5b^8}{ab^3}$. Assume that no denominator is equal to zero.

$$\frac{a^5b^8}{ab^3} = \left( \frac{a^5}{a} \right) \left( \frac{b^8}{b^3} \right)$$

Group powers that have the same base.

$$= (a^5 - 1), (b^8 - 3)$$

Quotient of Powers

1. Simplify $\frac{x^3y^4}{x^2y}$. Assume that no denominator is equal to zero.
Look for a pattern in the example below.

\[
\left( \frac{2}{5} \right)^3 = \left( \frac{2}{5} \right) \left( \frac{2}{5} \right) \left( \frac{2}{5} \right) = \frac{2 \cdot 2 \cdot 2}{5 \cdot 5 \cdot 5} \quad \text{or} \quad \frac{2^3}{5^3}
\]

3 factors 3 factors

**KEY CONCEPT**

**Power of a Quotient**

**Words** To find the power of a quotient, find the power of the numerator and the power of the denominator.

**Symbols** For any integer \( m \) and any real numbers \( a \) and \( b \), \( b \neq 0 \), \( \left( \frac{a}{b} \right)^m = \frac{a^m}{b^m} \).

---

**EXAMPLE**

**Power of a Quotient**

Simplify \( \left( \frac{2p^2}{3} \right)^4 \).

\[
\left( \frac{2p^2}{3} \right)^4 = \left( \frac{2}{3} \right)^4 \quad \text{Power of a Quotient}
\]

\[
= \left( \frac{2}{3} \right)^4 \quad \text{Power of a Product}
\]

\[
= \frac{16p^8}{81} \quad \text{Power of a Power}
\]

---

**Simplify each expression.**

2A. \( \left( \frac{3x^4}{4} \right)^3 \)

2B. \( \left( \frac{5x^5y}{6} \right)^2 \)

**Negative Exponents** A graphing calculator can be used to investigate expressions with 0 as an exponent and negative exponents.

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**Graphing Calculator Lab**

**Zero Exponent and Negative Exponents**

Use the \( ^\_x \) key to evaluate expressions with exponents.

1. Copy and complete the table.

<table>
<thead>
<tr>
<th>Power</th>
<th>2⁴</th>
<th>2³</th>
<th>2²</th>
<th>2¹</th>
<th>2⁰</th>
<th>2⁻¹</th>
<th>2⁻²</th>
<th>2⁻³</th>
<th>2⁻⁴</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. Describe the relationship between each pair of values.
   a. \( 2^4 \) and \( 2^{-4} \)  
   b. \( 2^3 \) and \( 2^{-3} \)  
   c. \( 2^2 \) and \( 2^{-2} \)  
   d. \( 2^1 \) and \( 2^{-1} \)

3. **Make a conjecture** as to the fractional value of \( 5^{-1} \). Verify your conjecture using a calculator.

4. What is the value of \( 5^0 \)?

5. What happens when you evaluate \( 0^0 \)?
To understand why a calculator gives a value of 1 for $2^0$, study the two methods used to simplify $\frac{2^4}{2^4}$.

**Method 1**
\[
\frac{2^4}{2^4} = 2^4 - 4 \quad \text{Quotient of Powers} \quad \frac{2^4}{2^4} = \frac{1 \cdot 1 \cdot 1 \cdot 1}{2 \cdot 2 \cdot 2 \cdot 2} \quad \text{Definition of powers}
\]
\[
= 2^0 \quad \text{Subtract} \quad = 1 \quad \text{Simplify}
\]

Since $\frac{2^4}{2^4}$ cannot have two different values, we can conclude that $2^0 = 1$.

**Example Zero Exponent**

Simplify each expression. Assume that no denominator is equal to zero.

a. \[
\left( -\frac{3x^5y}{8xy^7} \right)^0 = 1 \quad a^0 = 1
\]

b. \[
\frac{t^3s^0}{t} = t^3(1) \quad a^0 = 1
\]
\[
= \frac{t^3}{t} \quad \text{Simplify}
\]
\[
= t^2 \quad \text{Quotient of Powers}
\]

To investigate the meaning of a negative exponent, we can simplify expressions like $\frac{8^2}{8^5}$ in two ways.

**Method 1**
\[
\frac{8^2}{8^5} = 8^2 - 5 \quad \text{Quotient of Powers} \quad \frac{8^2}{8^5} = \frac{1 \cdot 1}{8 \cdot 8 \cdot 8 \cdot 8 \cdot 8} \quad \text{Definition of powers}
\]
\[
= 8^{-3} \quad \text{Subtract} \quad = \frac{1}{8^3} \quad \text{Simplify}
\]

Since $\frac{8^2}{8^5}$ cannot have two different values, we can conclude that $8^{-3} = \frac{1}{8^3}$.
An expression is simplified when it contains only positive exponents.

### Example: Negative Exponents

Simplify each expression. Assume that no denominator is equal to zero.

**a.** \( \frac{b^{-3}c^2}{d^{-5}} \)

\[
\frac{b^{-3}c^2}{d^{-5}} = \left( \frac{b^{-3}}{1} \right) \left( \frac{c^2}{1} \right) \left( \frac{1}{d^{-5}} \right)
\]

Write as a product of fractions.

\[
= \left( \frac{1}{b^3} \right) \left( \frac{c^2}{1} \right) \left( \frac{d^5}{1} \right)
\]

\[a^{-n} = \frac{1}{a^n}\]

\[c^2d^5 \]

Multiply fractions.

**b.** \( \frac{-3a^{-4}b^7}{21a^2b^7c^{-5}} \)

\[
\frac{-3a^{-4}b^7}{21a^2b^7c^{-5}} = \left( \frac{-3}{21} \right) \left( \frac{a^{-4}}{a^2} \right) \left( \frac{b^7}{b^7} \right) \left( \frac{1}{c^{-5}} \right)
\]

Group powers with the same base.

Quotient of Powers and Negative Exponent Properties

\[a^{-4} = \frac{1}{a^4}\]

\[\frac{1}{7} \cdot 2a^{-4 - 2}b^7 - 7c^5 \]

Simplify.

\[\frac{1}{7} \cdot \frac{1}{a^6} \cdot (1)c^5 \]

Negative Exponent and Zero Exponent Properties

Multiply fractions.

\[-\frac{c^5}{7a^6} \]

**c.** \( \frac{-3q^{-2}rs^4}{-12qr^{-3}s^{-5}} \)

\[
\frac{-3q^{-2}rs^4}{-12qr^{-3}s^{-5}} = \left( \frac{-3}{-12} \right) \left( \frac{q^{-2}}{q} \right) \left( \frac{r}{r^{-3}} \right) \left( \frac{s^4}{s^{-5}} \right)
\]

Group powers with the same base.

Simplify.

Negative Exponent Property

\[\frac{1}{4}q^{-3}r^4s^9 \]

\[= \frac{q^4s^9}{4q^3} \]

**4A.** \( \frac{r^{-5}s^4}{t^{-3}} \)

**4B.** \( \frac{24x^{-2}y^4}{-6x^{-3}y^{-2}z^{-1}} \)
Write the ratio of the area of the circle to the area of the square in simplest form.

A. \( \frac{\pi}{2} \)  
B. \( \frac{\pi}{4} \)  
C. \( \frac{2\pi}{1} \)  
D. \( \frac{\pi}{3} \)

Read the Test Item

A ratio is a comparison of two quantities. It can be written in fraction form.

Solve the Test Item

- Area of circle: \( \pi r^2 \)
- Length of square: diameter of circle or \( 2r \)
- Area of square: \( (2r)^2 \)

\[
\frac{\text{area of circle}}{\text{area of square}} = \frac{\pi r^2}{(2r)^2} \\
= \frac{\pi r^2}{4r^2} \quad \text{Substitute.}
\]

\[
= \frac{\pi}{4} \\
= \frac{\pi \cdot r^2}{4} \\
= \frac{\pi \cdot r^0}{4} \quad \text{Quotient of Powers}
\]

The answer is B.

Apply Properties of Exponents

Some problems can be solved using estimation. The area of the circle is less than the area of the square. Therefore, the ratio of the two areas must be less than 1. Use 3 as an approximate value for \( \pi \) to determine which of the choices is less than 1.

Example 1

(p. 367)

Example 2

(p. 367)

Example 3

(p. 369)

Example 4

(p. 370)

Simplify. Assume that no denominator is equal to zero.

1. \( \frac{7^8}{7^2} \)
2. \( \frac{x^8y^{12}}{x^2y^7} \)
3. \( \frac{5pq^7}{10p^6q^3} \)
4. \( \frac{2c^3d^3}{7z^2} \)
5. \( \left( \frac{4a^2b}{2c^3} \right)^2 \)
6. \( \left( \frac{3mn^3}{6n^2} \right)^2 \)
7. \( y^0(y^5)(y^{-9}) \)
8. \( \frac{(4m^{-3}n^5)^0}{mn} \)
9. \( \left( 3x^2y^5 \right)^0 \)
10. \( \frac{c^{-5}}{d^3g^{-8}} \)
11. \( \frac{(cd-2)^3}{(c^4d^9)^{-2}} \)
13. **STANDARDIZED TEST PRACTICE** Find the ratio of the volume of the cylinder to the volume of the sphere.

\[
\frac{\text{Volume of cylinder}}{\text{Volume of sphere}} = \frac{\pi r^2 h}{\frac{4}{3} \pi r^3}
\]

A: $\frac{1}{2}$  
B: $\frac{3}{4}$  
C: $\frac{4}{3}$  
D: $\frac{3}{2}$

**Exercise**

Simplify. Assume that no denominator is equal to zero.

14. $\frac{4^{12}}{4^2}$
15. $\frac{3^{13}}{3^7}$
16. $\frac{p^7 n^3}{p^4 n^2}$

17. $\frac{y^3 z^9}{yz^2}$
18. $\left(\frac{5b^4 n}{2a^6}\right)^2$
19. $\left(\frac{3m^7}{4x^3 y^3}\right)^4$

20. $\left(\frac{r^{-2} t^5}{r^{-1}}\right)^0$
21. $\left(\frac{4c^{-2} d}{b-2c^5 d^{-1}}\right)^0$
22. $6^{-2}$

23. $5^{-3}$
24. $\left(\frac{4}{5}\right)^{-2}$
25. $\left(\frac{3}{2}\right)^{-3}$

26. $n^2(p^{-4})(n^{-5})$
27. $\frac{28a^7 c^{-4}}{7a^3 b^6 c^{-8}}$
28. $x^3 y^0 x^{-7}$

29. The area of the rectangle is $24x^5 y^3$ square units. Find the length of the rectangle.
30. The area of the triangle is $100a^3 b$ square units. Find the height of the triangle.

Simplify. Assume that no denominator is equal to zero.

31. $\frac{-2a^3}{10a^8}$
32. $\frac{15b}{45b^5}$
33. $\frac{30h^{-2} k^{14}}{5h k^{-3}}$

34. $\frac{18x^3 y^2 z^7}{-2x^2 y z}$
35. $\frac{-19y^0 z^4}{-3z^{16}}$
36. $\left(\frac{5r^{-2}}{2r^3}\right)^2$

37. $\frac{p^{-4} q^{-3}}{(p^5 q^2)^{-1}}$
38. $\left(\frac{r^{-2} t^5}{t^{-1}}\right)^0$
39. $\left(\frac{5b^{-2} n^4}{n^2 z^{-3}}\right)^{-1}$

**PROBABILITY** For Exercises 40 and 41, use the following information.

If you toss a coin, the probability of getting heads is $\frac{1}{2}$. If you toss a coin 2 times, the probability of getting heads each time is $\left(\frac{1}{2}\right)^2$.

40. Write an expression to represent the probability of tossing a coin $n$ times and getting $n$ heads.
41. Express your answer to Exercise 40 as a power of 2.
**SOUND** For Exercises 42–44, use the following information.
The intensity of sound can be measured in watts per square meter. The table gives the watts per square meter for some common sounds.

<table>
<thead>
<tr>
<th>Watts per Square Meter</th>
<th>Common Sounds</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^2$</td>
<td>jet plane (30 m away)</td>
</tr>
<tr>
<td>$10^1$</td>
<td>pain level</td>
</tr>
<tr>
<td>$10^0$</td>
<td>amplified music (2 m away)</td>
</tr>
<tr>
<td>$10^{-2}$</td>
<td>noisy kitchen</td>
</tr>
<tr>
<td>$10^{-3}$</td>
<td>heavy traffic</td>
</tr>
<tr>
<td>$10^{-6}$</td>
<td>normal conversation</td>
</tr>
<tr>
<td>$10^{-7}$</td>
<td>average home</td>
</tr>
<tr>
<td>$10^{-9}$</td>
<td>soft whisper</td>
</tr>
<tr>
<td>$10^{-12}$</td>
<td>barely audible</td>
</tr>
</tbody>
</table>

42. How many times more intense is the sound from heavy traffic than the sound from normal conversation?

43. What sound is 10,000 times as loud as a noisy kitchen?

44. How does the intensity of a whisper compare to that of normal conversation?

**LIGHT** For Exercises 45 and 46, use the table at the right.

<table>
<thead>
<tr>
<th>Spectrum of Electromagnetic Radiation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Region</td>
</tr>
<tr>
<td>--------</td>
</tr>
<tr>
<td>Radio</td>
</tr>
<tr>
<td>Microwave</td>
</tr>
<tr>
<td>Infrared</td>
</tr>
<tr>
<td>Visible</td>
</tr>
<tr>
<td>Ultraviolet</td>
</tr>
<tr>
<td>X rays</td>
</tr>
<tr>
<td>Gamma Rays</td>
</tr>
</tbody>
</table>

45. Express the range of the wavelengths of visible light using positive exponents. Then evaluate each expression.

46. Express the range of the wavelengths of X rays using positive exponents. Then evaluate each expression.

47. **COMPUTERS** In 1993, the processing speed of a desktop computer was about $10^8$ instructions per second. By 2004, it had increased to $10^{10}$ instructions per second. How many times faster is the newer computer?

48. **OPEN ENDED** Name two monomials whose product is $54x^2y^3$.

49. **ALTERNATIVE METHODS** Describe a method of simplifying $\frac{a^3b^5}{ab^2}$ using negative exponents instead of the Quotient of Powers Property.

**CHALLENGE** Simplify. Assume that no denominator equals zero.

50. $a^2(a^3)$

51. $(5^{4x-3})(5^{2x+1})$

52. $\frac{c^x + 7}{c^x - 4}$

53. **REASONING** Write a convincing argument to show why $3^0 = 1$ using the following pattern: $3^5 = 243$, $3^4 = 81$, $3^3 = 27$, $3^2 = 9$. 

---

**Real-World Link**

Timbre is the quality of the sound produced by a musical instrument. Sound quality is what distinguishes the sound of a note played on a flute from the sound of the same note played on a trumpet with the same frequency and intensity.

Source: www.school.discovery.com
54. **FIND THE ERROR** Jamal and Angelina are simplifying \(-\frac{4x^3}{x^5}\). Who is correct? Explain your reasoning.

<table>
<thead>
<tr>
<th>Jamal</th>
<th>Angelina</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-\frac{4x^3}{x^5} = \frac{-4x^3}{x^{5}})</td>
<td>(-\frac{4x^3}{x^5} = \frac{x^{3-5}}{4})</td>
</tr>
<tr>
<td>(-\frac{4x^3}{x^5} = -4x^{-2})</td>
<td>(-\frac{x^{3-5}}{4} = \frac{x^{-2}}{4} = \frac{1}{4x^2})</td>
</tr>
</tbody>
</table>

55. **Writing in Math** Use the information about pH levels on page 366 to explain how you can use the properties of exponents to compare measures on the pH scale. Demonstrate an example comparing two pH levels using the properties of exponents.

56. How many times greater is the volume of the larger cube than the volume of the smaller cube?
   - A 2
   - B 4
   - C 8
   - D 16

57. **REVIEW** \(\triangle QRS\) is similar to \(\triangle TUV\). What is the length of \(UV\)?

58. \((m^3n)(mn^2)\)
59. \((3x^4y^3)(4x^4y)\)
60. \((a^3x^2)^4\)
61. \((3cd^5)^2\)
62. \([(2^3)^2]^2\)
63. \((-3ab)^3(2b^3)^2\)

**NUTRITION** For Exercises 64 and 65, use the following information.
Between the ages of 11 and 18, you should get at least 1200 milligrams of calcium each day. One ounce of mozzarella cheese has 147 milligrams of calcium, and one ounce of Swiss cheese has 219 milligrams. Suppose you want to eat no more than 8 ounces of cheese. (Lesson 6-8)

64. Draw a graph showing the possible amounts of each type of cheese you can eat and still get your daily requirement of calcium. Let \(x\) be the amount of mozzarella cheese and \(y\) be the amount of Swiss cheese.

65. List three possible solutions.

**PREREQUISITE SKILL** Evaluate each expression when \(a = 5\), \(b = -2\), and \(c = 3\). (Lesson 1-2)
66. \(5b^2\)
67. \(b^3 + 3ac\)
68. \(-2b^4 - 5b^3 - b\)
Mathematical Prefixes and Everyday Prefixes

You may have noticed that many prefixes used in mathematics are also used in everyday language. You can use the everyday meaning of these prefixes to better understand their mathematical meaning. The table shows four mathematical prefixes along with their meaning and an example of an everyday word using that prefix.

<table>
<thead>
<tr>
<th>Prefix</th>
<th>Everyday Meaning</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>mono-</td>
<td>1. one; single; alone</td>
<td>monologue A continuous series of jokes or comic stories delivered by one comedian.</td>
</tr>
<tr>
<td>bi-</td>
<td>1. two 2. both 3. both sides, parts, or directions</td>
<td>bicycle A vehicle with two wheels behind one another.</td>
</tr>
<tr>
<td>tri-</td>
<td>1. three 2. occurring at intervals of three 3. occurring three times during</td>
<td>trilogy A group of three dramatic or literary works related in subject or theme.</td>
</tr>
<tr>
<td>poly-</td>
<td>1. more than one; many; much</td>
<td>polygon A closed plane figure bounded by three or more line segments.</td>
</tr>
</tbody>
</table>

Source: The American Heritage Dictionary of the English Language

You can use your everyday understanding of prefixes to help you understand mathematical terms that use those prefixes.

Reading to Learn

1. Give an example of a geometry term that uses one of these prefixes. Then define that term.

2. **MAKE A CONJECTURE** Given your knowledge of the meaning of the word monomial, make a conjecture as to the meaning of each of the following mathematical terms.

   a. binomial
   b. trinomial
   c. polynomial

3. Research the following prefixes and their meanings.

   a. semi-
   b. hexa-
   c. octa-
   d. penta-
   e. tri-
   f. quad-
Algebra Lab
Polynomials

Algebra tiles can be used to model polynomials. A polynomial is a monomial or the sum of monomials. The diagram at the right shows the models.

<table>
<thead>
<tr>
<th>Polynomial Models</th>
</tr>
</thead>
<tbody>
<tr>
<td>Polynomials are modeled using three types of tiles.</td>
</tr>
<tr>
<td>Each tile has an opposite.</td>
</tr>
</tbody>
</table>

ACTIVITY

Use algebra tiles to model each polynomial.

- **4x**
  To model this polynomial, you will need 4 green \(x\)-tiles.

- **2x^2 - 3**
  To model this polynomial, you will need 2 blue \(x^2\)-tiles and 3 red \(-1\)-tiles.

- **−x^2 + 3x + 2**
  To model this polynomial, you will need 1 red \(-x^2\)-tile, 3 green \(x\)-tiles, and 2 yellow \(1\)-tiles.

MODEL AND ANALYZE

Use algebra tiles to model each polynomial. Then draw a diagram of your model.

1. \(-2x^2\)  
2. \(5x - 4\)
3. \(3x^2 - x\)  
4. \(x^2 + 4x + 3\)

Write an algebraic expression for each model.

5.
6.
7.
8.

9. **MAKE A CONJECTURE** Write a sentence or two explaining why algebra tiles are sometimes called area tiles.
Chapter 7
Polynomials

The number of hours \( H \) spent per person per year playing video games from 2000 through 2005 is shown in the table. These data can be modeled by the equation

\[
H = \frac{1}{4}(t^4 - 9t^3 + 24t^2 + 19t + 280),
\]

where \( t \) is the number of years since 2000. The expression \( t^4 - 9t^3 + 24t^2 + 19t + 280 \) is an example of a polynomial.

**Degree of a Polynomial** A polynomial is a monomial or a sum of monomials. Some polynomials have special names. A binomial is the sum of two monomials, and a trinomial is the sum of three monomials.

<table>
<thead>
<tr>
<th>Monomial</th>
<th>Binomial</th>
<th>Trinomial</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>3 + 4y</td>
<td>( x + y + z )</td>
</tr>
<tr>
<td>( 4ab^2c^2 )</td>
<td>7pq + pq^2</td>
<td>( 3v^2 - 2w + ab^3 )</td>
</tr>
</tbody>
</table>

**EXAMPLE** Identify Polynomials

State whether each expression is a polynomial. If it is a polynomial, identify it as a monomial, binomial, or trinomial.

1a. \( 2x - 3yz \)  
Yes, \( 2x - 3yz = 2x + (−3yz) \), the sum of two monomials.  
binomial

1b. \( 8n^3 + 5n^{-2} \)  
No, \( 5n^{-2} = \frac{5}{n^2} \), which is not a monomial.  
none of these

1c. \( −8 \)  
Yes, \( −8 \) is a real number.  
monomial

1d. \( 4a^2 + 5a + a + 9 \)  
Yes, the expression simplifies to \( 4a^2 + 6a + 9 \), so it is the sum of three monomials.  
trinomial

**CHECK Your Progress**

1A. \( x \)  
1B. \( −3y^2 - 2y + 4y - 1 \)
1C. \( 5rs + 7tuv \)  
1D. \( 10x^{-4} - 8x^3 \)
EXAMPLE 2 Write a Polynomial

GEOMETRY Write a polynomial to represent the area of the shaded region.

Words
The area of the shaded region is the area of the rectangle minus the area of the circle.

Variables
area of shaded region = \( A \)
width of rectangle = \( 2r \)
rectangle area = \( b(2r) \)
circle area = \( \pi r^2 \)

Equation
\[
A = b(2r) - \pi r^2
\]

The polynomial representing the area of the shaded region is \( 2br - \pi r^2 \).

2. Write a polynomial to represent the area of the shaded region.

The degree of a monomial is the sum of the exponents of all its variables.

The degree of a polynomial is the greatest degree of any term in the polynomial. To find the degree of a polynomial, you must find the degree of each term.

### Degree of a Polynomial

Find the degree of each polynomial.

<table>
<thead>
<tr>
<th>Polynomial</th>
<th>Terms</th>
<th>Degree of Each Term</th>
<th>Degree of Polynomial</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. ( 5mn^2 )</td>
<td>( 5mn^2 )</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>b. (-4x^2y^3 + 3x^2y^2 + 5)</td>
<td>(-4x^2y^3, 3x^2y^2, 5)</td>
<td>4, 2, 0</td>
<td>4</td>
</tr>
<tr>
<td>c. (3a + 7ab - 2a^2b + 16)</td>
<td>(3a, 7ab, -2a^2b, 16)</td>
<td>1, 2, 3, 0</td>
<td>3</td>
</tr>
</tbody>
</table>

3A. \( 7xy^2z \)  
3B. \( 12m^3n^2 - 8mn^2 + 3 \)  
3C. \( 2rs - 3rs^2 - 7r^2s^2 - 13 \)

Write Polynomials in Order The terms of a polynomial are usually arranged so that the powers of one variable are in ascending (increasing) order or descending (decreasing) order.
EXAMPLE

**Arrange Polynomials in Ascending Order**

Arrange the terms of each polynomial so that the powers of \( x \) are in ascending order.

a. \( 7x^2 + 2x^4 - 11 \)

\[
7x^2 + 2x^4 - 11 = 7x^2 + 2x^4 - 11x^0 \quad x^0 = 1
\]

\[
= -11 + 7x^2 + 2x^4 \quad \text{Compare powers of } x: 0 < 2 < 4.
\]

b. \( 2xy^3 + y^2 + 5x^3 - 3x^2y \)

\[
2xy^3 + y^2 + 5x^3 - 3x^2y = 2x^1y^3 + y^2 + 5x^3 - 3x^2y^1 \quad x = x^1
\]

\[
y^2 + 2xy^3 - 3x^2y + 5x^3 \quad \text{Compare powers of } x: 0 < 1 < 2 < 3.
\]

**4A.** \( 3x^2y^4 + 2x^4y^2 - 4x^3y + x^5 - y^2 \)

**4B.** \( 7x^3 - 4xy^4 + 3x^2y^3 - 11x^6y \)

**EXAMPLE

**Arrange Polynomials in Descending Order**

Arrange the terms of each polynomial so that the powers of \( x \) are in descending order.

a. \( 6x^2 + 5 - 8x - 2x^3 \)

\[
6x^2 + 5 - 8x - 2x^3 = 6x^2 + 5x^0 - 8x^1 - 2x^3 \quad x^0 = 1 \text{ and } x = x^1
\]

\[
= -2x^3 + 6x^2 - 8x + 5 \quad 3 > 2 > 1 > 0
\]

b. \( 3a^3x^2 - a^4 + 4ax^5 + 9a^2x \)

\[
3a^3x^2 - a^4 + 4ax^5 + 9a^2x = 3a^3x^2 - a^4x^0 + 4a^1x^5 + 9a^2x^1 \quad a = a^1, x^0 = 1, \text{ and } x = x^1
\]

\[
= 4ax^5 + 3a^3x^2 + 9a^2x - a^4 \quad 5 > 2 > 1 > 0.
\]

**5A.** \( 4x^2 + 2x^3y + 5 - x \)

**5B.** \( x + 2x^7y - 5x^4y^8 - x^2y^2 + 3 \)

**Check Your Understanding**

State whether each expression is a polynomial. If the expression is a polynomial, identify it as a **monomial**, a **binomial**, or a **trinomial**.

1. \( 5x - 3xy + 2x \)

2. \( \frac{2z}{5} \)

3. \( 9a^2 + 7a - 5 \)

**Example 2**

**GEOMETRY** Write a polynomial to represent the area of the shaded region.
Lesson 7-3 Polynomials

Find the degree of each polynomial.

5. 1
6. $3x + 2$
7. $2x^2y^3 + 6x^4$

Arrange the terms of each polynomial so that the powers of $x$ are in ascending order.

8. $6x^3 - 12 + 5x$
9. $-7a^2x^3 + 4x^2 - 2ax^5 + 2a$

Arrange the terms of each polynomial so that the powers of $x$ are in descending order.

10. $2c^5 + 9cx^2 + 3x$
11. $y^3 + x^3 + 3x^2y + 3xy^2$

State whether each expression is a polynomial. If the expression is a polynomial, identify it as a monomial, a binomial, or a trinomial.

12. 14
13. $\frac{6m^2}{p} + p^3$
14. $7b - 3.2c + 8b$
15. $\frac{1}{3}x^2 + x - 2$
16. $6gh^2 - 4g^2h + g$
17. $-4 + 2a + \frac{5}{a^2}$

GEOMETRY Write a polynomial to represent the area of each shaded region.

18.

19.

20.

21.

Find the degree of each polynomial.

22. $5x^3$
23. $9y$
24. $4ab$
25. $-13$
26. $c^4 + 7c^2$
27. $6n^3 - n^2p^2$
28. $15 - 8aq$
29. $3a^2b^3c^4 - 18a^5c$
30. $2x^3 - 4y + 7xy$
31. $3z^5 - 2x^2y^3z - 4x^2z$
32. $7 + d^5 - b^2c^2d^3 + b^6$
33. $11r^2t^4 - 2s^4t^5 + 24$

Arrange the terms of each polynomial so that the powers of $x$ are in ascending order.

34. $2x + 3x^2 - 1$
35. $9x^3 + 7 - 3x^5$
36. $c^2x^3 - c^3x^2 + 8c$
37. $x^3 + 4a + 5a^2x^6$
38. $4 + 3ax^5 + 2ax^2 - 5a^7$
39. $10x^3y^2 - 3x^9y + 5y^4 + 2x^2$
40. $3xy^2 - 4x^3 + x^2y + 6y$
41. $-8a^5x + 2ax^4 - 5 - a^2x^2$
Arrange the terms of each polynomial so that the powers of $x$ are in descending order.

42. $5 + x^5 + 3x^3$
43. $2x - 1 + 6x^2$
44. $4a^3x^2 - 5a + 2a^2x^3$
45. $b^2 + x^2 - 2xb$
46. $x^2 + cx^3 - 5c^3x^2 + 11x$
47. $9x^2 + 3 + 4ax^3 - 2a^2x$
48. $4x^3 + 3xy^4 - x^2y^3 + y^4$

50. **MONEY** Write a polynomial to represent the value of $q$ quarters, $d$ dimes and $n$ nickels.

51. **MULTIPLE BIRTHS** The rate of quadruplet births $Q$ in the United States in recent years can be modeled by $Q = -0.5t^3 + 11.7t^2 - 21.5t + 218.6$, where $t$ represents the number of years since 1992. For what values of $t$ does this model no longer give realistic data? Explain your reasoning.

52. Write a polynomial that represents the volume of the container.

53. If the height of the container is 6 inches and the radius is 2 inches, find the volume of the container.

54. Write two polynomials that represent the perimeter and area of the rectangle shown at right.

55. **OPEN ENDED** Give an example of a monomial of degree zero.

56. **REASONING** Explain why a polynomial cannot contain a variable term with a negative power.

57. **CHALLENGE** Tell whether the following statement is true or false. Explain your reasoning.

The degree of a binomial can never be zero.

58. **REASONING** Determine whether each statement is true or false. If false, give a counterexample.
   a. All binomials are polynomials.
   b. All polynomials are monomials.
   c. All monomials are polynomials.

59. **Writing in Math** Use the information about video game usage on page 376 to explain how polynomials can be useful in modeling data. Include a discussion of the accuracy of the equation by evaluating the polynomial for $t = \{0, 1, 2, 3, 4, 5\}$ and an example of how and why someone might use this equation.

---

**Real-World Link**

From 1980 to 1999, the number of triplet and higher births rose approximately 532% (from 1377 to 7321 births). This steep climb in multiple births coincides with the increased use of fertility drugs.

*Source: National Center for Health and Statistics*
60. Which expression could be used to represent the area of the shaded region of the rectangle, reduced to simplest terms?

A $x^2 + 3x$  
B $4x^2 + 8x$  
C $3x^2 - 7x$  
D $4x^2 + 7x$

61. **REVIEW** Lawanda rolled a six-sided game cube 30 times and recorded her results in the table below. Each side of the cube is a different color. Which color has the same experimental probability and theoretical probability?

<table>
<thead>
<tr>
<th>Color</th>
<th>Rolls</th>
</tr>
</thead>
<tbody>
<tr>
<td>Red</td>
<td>8</td>
</tr>
<tr>
<td>Blue</td>
<td>4</td>
</tr>
<tr>
<td>White</td>
<td>6</td>
</tr>
<tr>
<td>Orange</td>
<td>3</td>
</tr>
<tr>
<td>Green</td>
<td>5</td>
</tr>
<tr>
<td>Purple</td>
<td>5</td>
</tr>
</tbody>
</table>

62. $a^0b^{-2}c^{-1}$

63. $-\frac{5n^5}{n^8}$

64. $\left(\frac{4x^3y^2}{3z}\right)^2$

65. $\frac{(-y)^5m^8}{y^3m^{-7}}$

Determine whether each expression is a monomial. Write *yes* or *no*. (Lesson 7-1)

66. $3a + 4b$  
67. $\frac{6}{n}$  
68. $\frac{5}{3}$

Determine whether each relation is a function. (Lesson 3-6)

69.

70.

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>-2</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>-2</td>
</tr>
</tbody>
</table>

71. **MAPS** The scale of a road map is 1.5 inches = 100 miles. The distance between New Hartford, Connecticut, and Westerly, Rhode Island, by highway on the map is about 1.0 inch. What is the distance between these two cities? (Lesson 2-6)

Find each square root. Round to the nearest hundredth if necessary. (Lesson 1-8)

72. $\pm \sqrt{121}$  
73. $\sqrt{3.24}$  
74. $-\sqrt{52}$

**PREREQUISITE SKILL** Simplify each expression, if possible. If not possible, write in simplest form. (Lesson 1-5)

75. $3n + 5n$  
76. $9a^2 + 3a - 2a^2$  
77. $-3a + 5b + 4a - 7b$  
78. $4x + 3y - 6 + 7x + 8 - 10y$
Algebra Lab

Adding and Subtracting Polynomials

Monomials such as $5x$ and $-3x$ are called *like terms* because they have the same variable to the same power. When you use algebra tiles, you can recognize like terms because these tiles have the same size and shape as each other.

### Activity 1

Use algebra tiles to find $(3x^2 - 2x + 1) + (x^2 + 4x - 3)$.

**Step 1**

Model each polynomial.

- $3x^2 - 2x + 1 ightarrow \begin{array}{c}
        x^2 \\
        x^2 \\
        x^2 \\
        3x^2 \\
        -2x \\
        1 \\
    \end{array}$

- $x^2 + 4x - 3 ightarrow \begin{array}{c}
        x^2 \\
        x^2 \\
        x^2 \\
        x^2 \\
        4x \\
        -3 \\
    \end{array}$

**Step 2**

Combine like terms and remove zero pairs.

**Step 3**

Write the polynomial for the tiles that remain.

$(3x^2 - 2x + 1) + (x^2 + 4x - 3) = 4x^2 + 2x - 2$
Step 1  Model the polynomial $5x + 4$.

Step 2  To subtract $-2x + 3$, you must remove $2 -x$-tiles and 3 1-tiles. You can remove the 1-tiles, but there are no $-x$-tiles. Add 2 zero pairs of $x$-tiles. Then remove the 2 $-x$-tiles.

Step 3  The remaining tiles model $7x + 1$.

Recall that you can subtract a number by adding its additive inverse or opposite. Similarly, you can subtract a polynomial by adding its opposite.

**ACTIVITY 3** Use algebra tiles and the additive inverse, or opposite, to find $(5x + 4) - (-2x + 3)$.

Step 1  To find the difference of $5x + 4$ and $-2x + 3$, add $5x + 4$ and the opposite of $-2x + 3$. The opposite of $-2x + 3$ is $2x - 3$.

Step 2  Write the polynomial for the tiles that remain.

$(5x + 4) - (-2x + 3) = 7x + 1$

Notice that this is the same answer as in Activity 2.

**MODEL AND ANALYZE**

Use algebra tiles to find each sum or difference.

1. $(5x^2 + 3x - 4) + (2x^2 - 4x + 1)$
2. $(2x^2 + 5) + (3x^2 + 2x + 6)$
3. $(-4x^2 + x) + (5x - 2)$
4. $(3x^2 + 4x + 2) - (x^2 - 5x - 5)$
5. $(-x^2 + 7x) - (-x^2 + 3x)$
6. $(8x + 4) - (6x^2 + x - 3)$
7. Find $(2x^2 - 3x + 1) - (2x + 3)$ using each method from Activities 2 and 3. Illustrate and explain how zero pairs are used in each case.
From 2000 to 2003, the amount of sales (in millions of dollars) of rap/hip-hop music \( R \) and country music \( C \) in the United States can be modeled by the following equations, where \( t \) is the number of years since 2000.

\[
R = -132.32t^3 + 624.74t^2 - 773.61t + 1847.67 \\
C = -3.42t^3 + 8.6t^2 - 94.95t + 1532.56
\]

The total music sales \( T \) of rap/hip-hop and country music is \( R + C \).

**Add Polynomials** To add polynomials, you can group like terms horizontally or write them in column form, aligning like terms. Adding polynomials involves adding like terms.

**EXAMPLE**

**Add Polynomials**

Find \((3x^2 - 4x + 8) + (2x - 7x^2 - 5)\).

Method 1  Horizontal

\[
(3x^2 - 4x + 8) + (2x - 7x^2 - 5) = \left(3x^2 + (-7x^2)\right) + (-4x + 2x) + [8 + (-5)] \\
= -4x^2 - 2x + 3
\]

Method 2  Vertical

\[
\begin{align*}
3x^2 & - 4x + 8 \\
+ & -7x^2 + 2x - 5 \\
= & -4x^2 - 2x + 3
\end{align*}
\]

**Subtract Polynomials** Recall that you can subtract a real number by adding its opposite or additive inverse. Similarly, you can subtract a polynomial by adding its additive inverse.

To find the additive inverse of a polynomial, replace each term with its additive inverse.

<table>
<thead>
<tr>
<th>Polynomial</th>
<th>Additive Inverse</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-5m + 3n)</td>
<td>(5m - 3n)</td>
</tr>
<tr>
<td>(-2y^2 + 6y - 11)</td>
<td>(2y^2 - 6y + 11)</td>
</tr>
<tr>
<td>(7a + 9b - 4)</td>
<td>(-7a - 9b + 4)</td>
</tr>
</tbody>
</table>
**EXAMPLE**  Subtract Polynomials

Find \((3n^2 + 13n^3 + 5n) - (7n + 4n^3)\).

**Method 1**  Horizontal

Subtract \(7n + 4n^3\) by adding its additive inverse.

\[
(3n^2 + 13n^3 + 5n) - (7n + 4n^3) = (3n^2 + 13n^3 + 5n) + (-7n - 4n^3)
\]

The additive inverse of \(7n + 4n^3\) is \(-7n - 4n^3\).

\[
= 3n^2 + [13n^3 + (-4n^3)] + [5n + (-7n)]
\]

Group like terms.

\[
= 3n^2 + 9n^3 - 2n
\]

Combine like terms.

**Method 2**  Vertical

Align like terms in columns and subtract by adding the additive inverse.

\[
\begin{array}{c}
3n^2 + 13n^3 + 5n \\
(-) \quad 4n^3 + 7n
\end{array}
\]

Add the opposite.

\[
\begin{array}{c}
3n^2 + 13n^3 + 5n \\
(+) \quad -4n^3 - 7n
\end{array}
\]

\[
\begin{array}{c}
3n^2 + 9n^3 - 2n
\end{array}
\]

Thus, \((3n^2 + 13n^3 + 5n) - (7n + 4n^3) = 3n^2 + 9n^3 - 2n\) or, arranged in descending order, \(9n^3 + 3n^2 - 2n\).

2. Find \((4x^3 - 3x^2 + 6x - 4) - (-2x^3 + x^2 - 2)\).

When polynomials are used to model real-world data, their sums and differences can have real-world meaning, too.

**EDUCATION**  The total number of public school teachers \(T\) consists of two groups, elementary \(E\) and secondary \(S\). From 1992 through 2003, the number (in thousands) of secondary teachers and total teachers could be modeled by the following equations, where \(n\) is the number of years since 1992.

\[
S = 29n + 949 \quad T = 58n + 2401
\]

a. Find an equation that models the number of elementary teachers \(E\) for this time period.

Subtract the polynomial for \(S\) from the polynomial for \(T\).

\[
\begin{array}{c|c|c}
\text{Total} & 58n + 2401 \\
- \text{Secondary} & (-) 29n + 949 & \text{Add the opposite} \\
\hline
\text{Elementary} & 58n + 2401 \\
& (-) -29n - 949 & \text{Add the opposite} \\
& \hline & 29n + 1452
\end{array}
\]

An equation is \(E = 29n + 1452\).

b. Use the equation to predict the number of elementary teachers in 2015.

The year 2015 is 2015 \(-\) 1992 or 23 years after the year 1992.

If this trend continues, the number of elementary teachers in 2015 would be \(29(23) + 1452\) or about 2,119,000.
3. **WIRELESS DEVICES** An electronics store sells cell phones and pagers. The equations below represent the monthly sales \(m\) of cell phones \(C\) and pagers \(P\). Write an equation that represents the total monthly sales \(T\) of wireless devices. Use the equation to predict the number of wireless devices sold in 10 months.

\[
C = 7m + 137 \quad P = 4m + 78
\]

**Exercises**

**Examples 1, 2** (pp. 384, 385)

Find each sum or difference.

1. \((4p^2 + 5p) + (-2p^2 + p)\)
2. \((5y^2 - 3y + 8) + (4y^2 - 9)\)
3. \((8cd - 3d + 4c) + (-6 + 2cd)\)
4. \((-8xy + 3x^2 - 5y) + (4x^2 - 2y + 6xy)\)
5. \((6a^2 + 7a - 9) - (-5a^2 + a - 10)\)
6. \((g^3 - 2g^2 + 5g + 6) - (g^2 + 2g)\)
7. \((3ax^2 - 5x - 3a) - (6a - 8a^2x + 4x)\)
8. \((4rst - 8r^2s + s^2) - (6rs^2 + 5rst - 2s^2)\)

**Example 3** (p. 385)

**POPULATION** For Exercises 9 and 10, use the following information.

From 1980 through 2003, the female population \(F\) and the male population \(M\) of the United States (in thousands) are modeled by the following equations, where \(n\) is the number of years since 1980.

\[
F = 1,379n + 115,513 \quad M = 1,450n + 108,882
\]

9. Find an equation that models the total population \(T\) of the United States in thousands for this time period.

10. If this trend continues, what will the population of the U. S. be in 2010?

**Exercises**

Find each sum or difference.

11. \((6n^2 - 4) + (-2n^2 + 9)\)
12. \((9z - 3z^2) + (4z - 7z^2)\)
13. \((3 + a^2 + 2a) + (a^2 - 8a + 5)\)
14. \((-3n^2 - 8 + 2n) + (5n + 13 + n^2)\)
15. \((x + 5) + (2y + 4x - 2)\)
16. \((2b^3 - 4b + b^2) + (-9b^2 + 3b^3)\)
17. \((11 + 4d^2) - (3 - 6d^2)\)
18. \((4g^3 - 5g) - (2g^3 + 4g)\)
19. \((-4y^3 - y + 10) - (4y^3 + 3y^2 - 7)\)
20. \((4x + 5xy + 3y) - (3y + 6x + 8xy)\)
21. \((3x^2 + 8x + 4) - (5x^2 - 4)\)
22. \((5ab^2 + 3ab) - (2ab^2 + 4 - 8ab)\)

**GEOMETRY** The measures of two sides of a triangle are given. If \(P\) is the perimeter, find the measure of the third side.

23. \(P = 7x + 3y\)
24. \(P = 10x^2 - 5x + 16\)
Find each sum or difference.

25. \((3a + 2b - 7c) + (6b - 4a + 9c) + (-7c - 3a - 2b)\)

26. \((5x^2 - 3) + (x^2 - x + 11) + (2x^2 - 5x + 7)\)

27. \((3y^2 - 8) + (5y + 9) - (y^2 + 6y - 4)\)

28. \((9x^3 + 3x - 13) - (6x^2 - 5x) + (2x^3 - x^2 - 8x + 4)\)

**MOVIES** For Exercises 29 and 30, use the following information.

From 1995 to 2004, the number of indoor movie screens \(I\) and total movie screens \(T\) in the U.S. could be modeled by the following equations, where \(n\) is the number of years since 1995.

\[
I = -194.8n^2 + 2,658n + 26,933 \\
T = -193n^2 + 2,616n + 27,793
\]

29. Find an equation that models the number of outdoor movie screens \(D\).

30. If this trend continues, how many outdoor screens will there be in 2010?

**POSTAL SERVICE** For Exercises 31–33, use the following information.

The U.S. Postal Service restricts the sizes of boxes shipped by parcel post. The sum of the length and the girth of the box must not exceed 108 inches.

Suppose you want to make an open box using a 60-by-40-inch piece of cardboard by cutting squares out of each corner and folding up the flaps. The lid will be made from another piece of cardboard. You do not know how big the squares should be, so for now call the length of the side of each square \(x\).

31. Write polynomials to represent the length, width, and girth of the box formed.

32. Write and solve an inequality to find the least possible value of \(x\) you could use in designing this box so it meets postal regulations.

33. What is the greatest integral value of \(x\) you could use to design this box if it does not have to meet regulations?

**H.O.T. Problems:**

34. **REASONING** Explain why \(5xy^2\) and \(3x^2y\) are not like terms.

35. **OPEN ENDED** Write two polynomials with a difference of \(2x^2 + x + 3\).

36. **FIND THE ERROR** Esteban and Kendra are finding \((5a - 6b) - (2a + 5b)\). Who is correct? Explain your reasoning.

**Esteban**

\[
(5a - 6b) - (2a + 5b) \\
= (-5a + 6b) + (-2a - 5b) \\
= -7a + b
\]

**Kendra**

\[
(5a - 6b) - (2a + 5b) \\
= (5a - 6b) + (-2a - 5b) \\
= 3a - 11b
\]
**CHALLENGE** For Exercises 37–39, suppose \( x \) is an integer.

37. Write an expression for the next integer greater than \( x \).
38. Show that the sum of two consecutive integers, \( x \) and the next integer after \( x \), is always odd. (*Hint:* A number is considered even if it is divisible by 2.)
39. What is the least number of consecutive integers that must be added together to always arrive at an even integer?

40. **Writing in Math** Use the information about music sales on page 384 to explain how you can use polynomials to model sales. Include an equation that models total music sales, and an example of how and why someone might use this equation in your answer.

41. The perimeter of the rectangle shown below is \( 16a + 2b \). Which expression represents the width of the rectangle?

   ![Rectangle Diagram]

   A. \( 3a + 2b \)  
   B. \( 10a + 2b \)  
   C. \( 2a - 3b \)  
   D. \( 6a + 4b \)

42. **REVIEW** The scale factor of two similar polygons is 4:5. The perimeter of the larger polygon is 200 inches. What is the perimeter of the smaller polygon?

   F. 250 inches  
   H. 80 inches  
   G. 160 inches  
   J. 40 inches

**Spiral Review**

Find the degree of each polynomial. (*Lesson 7-3*)

43. \( 15t^3y^2 \)  
44. 24  
45. \( m^2 + n^3 \)  
46. \( 4x^2y^3z - 5x^3z \)

Simplify. Assume no denominator is equal to zero. (*Lesson 7-2*)

47. \( \frac{49a^4b^7c^2}{7a^3b^4c^2} \)
48. \( \frac{-4n^3p^5}{n^2} \)
49. \( \frac{(8n^7)^2}{(3n^2)^3} \)

**KEYBOARDING** For Exercises 50–53, use the table that shows keyboarding speeds of 12 students in words per minute (wpm) and weeks of experience. (*Lesson 4-7*)

<table>
<thead>
<tr>
<th>Experience (weeks)</th>
<th>4</th>
<th>7</th>
<th>8</th>
<th>1</th>
<th>6</th>
<th>3</th>
<th>5</th>
<th>2</th>
<th>9</th>
<th>6</th>
<th>7</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Keyboarding Speed (wpm)</td>
<td>33</td>
<td>45</td>
<td>46</td>
<td>20</td>
<td>40</td>
<td>30</td>
<td>38</td>
<td>22</td>
<td>52</td>
<td>44</td>
<td>42</td>
<td>55</td>
</tr>
</tbody>
</table>

50. Make a scatter plot of these data. Then draw a line of fit.
51. Find the equation of the line.
52. Use the equation to predict the speed of a student after a 12-week course.
53. Why is this equation not used to predict the speed for any number of weeks of experience?

**PREREQUISITE SKILL.** Simplify. (*Lesson 1-5*)

54. \( 6(3x - 8) \)  
55. \( -2(b + 9) \)  
56. \( -7(-5p + 4q) \)

57. \( 9(3a + 5b - c) \)  
58. \( 8(x^2 + 3x - 4) \)  
59. \( -3(2a^2 - 5a + 7) \)
Chapter 7
Mid-Chapter Quiz
Lessons 7-1 through 7-4

Simplify. (Lesson 7-1)
1. \(n^3(n^4)(n)\)
2. \(4ad(3a^3d)\)
3. \((-2w^3z^4)^3(-4wz^3)^2\)

4. MULTIPLE CHOICE Ruby says that \((xy)^2 = x^2 + 2xy + y^2\) for every value of \(x\) and \(y\), but Ebony disagrees. What does \((xy)^2\) really equal? (Lesson 7-1)
   A 2\(x^2y\)
   B 2\(xy\)
   C \(xy^2\)
   D \(x^2y^2\)

5. MULTIPLE CHOICE Which expression represents the volume of the cube? (Lesson 7-1)
   F 15\(x^3\)
   G 25\(x^2\)
   H 25\(x^3\)
   J 125\(x^3\)

Simplify. Assume that no denominator is equal to zero. (Lesson 7-2)
6. \(\frac{25p^{10}}{15p^3}\)
7. \(\left(\frac{6k^3}{7np^4}\right)^2\)
8. \(\frac{4x^0y^2}{(3y^{-3}z^5)^{-2}}\)
9. \(\frac{(m^2np^3)^{-3}}{(m^5n^3p^6)^{-4}}\)

10. GEOMETRY The area of the rectangle is 66\(a^3b^5c^7\) square inches. Find the length of the rectangle. (Lesson 7-2)

11. MULTIPLE CHOICE The wavelength of a microwave is \(10^{-2}\) centimeters, and the wavelength of an X ray is \(10^{-8}\) centimeters. How many times greater is the length of a microwave than an X ray? (Lesson 7-3)
   A 10^10
   B 10^6
   C 10^-6
   D 10^-10

Find the degree of each polynomial. (Lesson 7-3)
12. 5\(x^4\)
13. \(-9n^3p^4\)
14. 7\(a^2 - 2ab^2\)
15. \(-6 - 8x^2y^2 + 5y^3\)

GEOMETRY For Exercises 16 and 17, use the figure below. (Lesson 7-3)

16. Write a polynomial that represents the area of the figure.
17. If \(x = 7\) feet, \(y = 3\) feet, and \(z = 2\) feet, find the area of the figure.

Arrange the terms of each polynomial so that the powers of \(x\) are in ascending order. (Lesson 7-4)
18. 4\(x^2 + 9x - 12 + 5x^3\)
19. 2\(xy^4 + x^3y^5 + 5x^5y - 13x^2\)

20. MULTIPLE CHOICE If three consecutive integers are \(x\), \(x + 1\), and \(x + 2\), what is the sum of these three integers? (Lesson 7-4)
   F 2\(x + 3\)
   G 3\(x + 3\)
   H \(x(x + 1)(x + 2)\)
   J \(x^3 + 3x^2 + 2x\)
Multiplying a Polynomial by a Monomial

Main Ideas
- Find the product of a monomial and a polynomial.
- Solve equations involving polynomials.

Product of Monomial and Polynomial

The Distributive Property can be used to multiply a polynomial by a monomial.

EXAMPLE

Multiply a Polynomial by a Monomial

Find \(-2x^2(3x^2 - 7x + 10)\).

Method 1 Horizontal

\[
-2x^2(3x^2 - 7x + 10) = -2x^2(3x^2) - (-2x^2)(7x) + (-2x^2)(10) \\
= -6x^4 + 14x^3 - 20x^2
\]

Method 2 Vertical

\[
\begin{array}{c}
3x^2 - 7x + 10 \\
\times \quad -2x^2 \\
\hline
-6x^4 + 14x^3 - 20x^2
\end{array}
\]

Check Your Progress

1. Find \(5a^2(-4a^2 + 2a - 7)\).

EXAMPLE

Simplify Expressions

Simplify \(4(3d^2 + 5d) - d(d^2 - 7d + 12)\).

\[
4(3d^2 + 5d) - d(d^2 - 7d + 12) \\
= 4(3d^2) + 4(5d) - (d)(d^2) + (d)(7d) + (-d)(12) \\
= 12d^2 + 20d - d^3 + 7d^2 - 12d \\
= 12d^2 + 20d - 3 + 7d^2 - 12d \\
= -d^3 + (12d^2 + 7d^2) + (20d - 12d) \\
= -d^3 + 19d^2 + 8d
\]

Check Your Progress

2. Simplify \(3(5x^2 + 2x - 4) - x(7x^2 + 2x - 3)\).
**PHONE SERVICE** Greg pays a fee of $20 a month for local calls. Long-distance rates are 6¢ per minute for in-state calls and 5¢ per minute for out-of-state calls. Suppose Greg makes 300 minutes of long-distance phone calls in January and $m$ of those minutes are for in-state calls.

**a. Find an expression for Greg’s phone bill for January.**

<table>
<thead>
<tr>
<th>Words</th>
<th>Variables</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bill = service fee + in-state minutes • 6¢ per minute + out-of-state minutes • 5¢ per minute.</td>
<td>$m$ = number of minutes of in-state calls, then $300 - m$ = number of minutes of out-of-state calls. Let $B =$ phone bill for the month of January.</td>
<td>$B = 20 + m \cdot 0.06 + (300 - m) \cdot 0.05$</td>
</tr>
</tbody>
</table>

Write the equation.

$= 20 + 0.06m + 300(0.05) - m(0.05)$

Distributive Property

$= 20 + 0.06m + 15 - 0.05m$

Simplify.

$= 35 + 0.01m$

Simplify.

Greg’s bill for January is $35 + 0.01m$, for $m$ minutes of in-state calls.

**b. Evaluate the expression to find the cost if Greg had 37 minutes of in-state calls in January.**

$35 + 0.01m = 35 + 0.01(37)
\quad m = 37$

$= 35 + 0.37$

Multiply.

$= 35.37$

Add.

Greg’s bill was $35.37.

**3. A parking garage charges $30 per month plus $0.50 per daytime hour and $0.25 per hour during nights and weekends. Suppose Juana parks in the garage for 47 hours in January and $h$ of those are night and weekend hours. Find an expression for her January bill. Then find the cost if Juana had 12 hours of night and weekend hours.**

**Solve Equations with Polynomial Expressions** Many equations contain polynomials that must be added, subtracted, or multiplied.

**EXAMPLE**

**Polynomials on Both Sides**

**Solve** $y(y - 12) + y(y + 2) + 25 = 2y(y + 5) - 15$.

$y(y - 12) + y(y + 2) + 25 = 2y(y + 5) - 15$ Original equation

$y^2 - 12y + y^2 + 2y + 25 = 2y^2 + 10y - 15$ Distributive Property

$2y^2 - 10y + 25 = 2y^2 + 10y - 15$ Combine like terms.

$-10y + 25 = 10y - 15$ Subtract $2y^2$ from each side.

$-20y + 25 = -15$ Subtract $10y$ from each side.

$-20y = -40$ Subtract 25 from each side.

$y = 2$ Divide each side by $-20$. 

**Extra Examples at algebra1.com**
**CHECK Your Progress**

Find each product.

1. \(-3y(5y + 2)\)
2. \(9b^2(2b^3 - 3b^2 + b - 8)\)
3. \(2x(4n^4 - 3ax + 6x^2)\)
4. \(-4xy(5x^2 - 12xy + 7y^2)\)

**Example 2**

Simplify.

5. \(t(5t - 9) - 2t\)
6. \(x(3x + 4) + 2(7x - 3)\)
7. \(5m(4r^3 + 6n^2 - 2n + 3) - 4(n^2 + 7n)\)
8. \(4y^2(y^2 - 2y + 5) + 3y(2y^2 - 2)\)

**Example 3**

**SAVINGS** For Exercises 9–11, use the following information.
Matthew’s grandmother left him $10,000 for college. Matthew puts some of the money into a savings account earning 3% interest per year. With the rest, he buys a certificate of deposit (CD) earning 5% per year.

9. If Matthew puts \(x\) dollars into the savings account, write an expression to represent the amount of the CD.
10. Write an equation for the total amount of money \(T\) Matthew will have saved for college after one year.
11. If Matthew puts $3000 in savings, how much money will he have in one year?

**Example 4**

Solve each equation.

12. \(-2(w + 1) + w = 7 - 4w\)
13. \(3(y - 2) + 2y = 4y + 14\)
14. \(a(a + 3) + a(a - 6) + 35 = a(a - 5) + a(a + 7)\)
15. \(n(n - 4) + n(n + 8) = n(n - 13) + n(n + 1) + 16\)

**Exercises**

Find each product.

16. \(r(5r + r^2)\)
17. \(w(2w^3 - 9w^2)\)
18. \(-4x(8 + 3x)\)
19. \(5y(-2y^2 - 7y)\)
20. \(7ag(g^3 + 2ag)\)
21. \(-3np(n^2 - 2p)\)
22. \(-2b^2(3b^2 - 4b + 9)\)
23. \(6x^3(5 + 3x - 11x^2)\)
24. \(8x^2y(5x + 2y^2 - 3)\)
25. \(-cd^2(3d + 2c^2d - 4c)\)

Simplify.

26. \(d(-2d + 4) + 15d\)
27. \(-x(4x^2 - 2x) - 5x^3\)
28. \(3w(6w - 4) + 2(w^2 - 3w + 5)\)
29. \(5n(2n^3 + n^2 + 8) + n(4 - n)\)
30. \(10(4m^3 - 3m + 2) - 2m(-3m^2 - 7m + 1)\)
31. \(4y(y^2 - 8y + 6) - 3(2y^3 - 5y^2 + 2)\)
**SAVINGS** For Exercises 32 and 33, use the following information. Marta has $6000 to invest. She puts $x$ dollars of this money into a savings account that earns 2% interest per year. With the rest, she buys a certificate of deposit that earns 4% per year.

32. Write an equation for the amount of money $T$ Marta will have in one year.

33. Suppose at the end of one year, Marta has a total of $6210. How much money did Marta invest in each account?

**FARMING** A farmer plants corn in a field with a length to width ratio of 5:4. Next year, he plans to increase the field’s area by increasing its length by 12 feet. Write an expression for this new area.

**CLASS TRIP** Mr. Wong’s American History class will take taxis from their hotel in Washington, D.C., to the Lincoln Memorial. The fare is $2.75 for the first mile and $1.25 for each additional mile. If the distance is $m$ miles and $t$ taxis are needed, write an expression for the cost to transport the group.

**Solve each equation.**

36. $2(4x - 7) = 5(-2x - 9) - 5$
37. $4(3p + 9) - 5 = -3(12p - 5)$
38. $d(d - 1) + 4d = d(d - 8)$
39. $c(c + 3) - c(c - 4) = 9c - 16$
40. $a(3a - 2) + 2a(a + 4) = a(a + 2) + 4a(a - 3) + 48$
41. $3(4w - 2) + 6(w + 4) - 3 = 4w - 7(w + 2) + 5(3w + 7)$

**Expand and simplify.**

42. $4(x + 2) - 6$
43. $3x - 2(x + 1)$

**Find each product.**

44. $-\frac{3}{4}hk^2(20k^2 + 5h - 8)$
45. $\frac{2}{3}a^2b(6a^3 - 4ab + 9b^2)$
46. $-5a^3b(2b + 5ab - b^2 + a^3)$
47. $4p^2q^2(2p^2 - q^2 + 9p^3 + 3q)$

**Simplify.**

48. $-3c^2(2c + 7) + 4c(3c^2 - c + 5) + 2(c^2 - 4)$
49. $4x^2(x + 2) + 3x(5x^2 + 2x - 6) - 5(3x^2 - 4x)$

**Solve each equation.**

50. $2n(n + 4) + 18 = n(n + 5) + n(n - 2) - 7$
51. $3g(g - 4) - 2g(g - 7) = g(g + 6) - 28$

**GEOMETRY** Find the area of each shaded region in simplest form.

52. $4x$
53. $5p$

**Real-World Link**

Inside the Lincoln Memorial is a 19-foot marble statue of the United States’ 16th president. The statue is flanked on either side by the inscriptions of Lincoln’s Second Inaugural Address and Gettysburg Address.

*Source: washington.org*
**VOLUNTEERING** For Exercises 54 and 55, use the following information.
Loretta is making baskets of apples and oranges for homeless shelters. She wants to place a total of 10 pieces of fruit in each basket. Apples cost 25¢ each, and oranges cost 20¢ each.

54. If \( a \) represents the number of apples Loretta uses, write a polynomial model in simplest form for the total amount of money \( T \) Loretta will spend.
55. If Loretta uses 4 apples in each basket, find the total cost for fruit.

**SALES** For Exercises 56 and 57, use the following information.
A store advertises that all sports equipment is 30% off the retail price. In addition, the store asks customers to select and pop a balloon to receive a coupon for an additional \( n \) percent off one of their purchases.

56. Write an expression for the cost of a pair of inline skates with retail price \( p \).
57. Use this expression to calculate the cost, not including sales tax, of a $200 pair of inline skates for an additional 10% off.

58. **SPORTS** You may have noticed that when runners race around a curved track, their starting points are staggered. This is so each contestant runs the same distance to the finish line.

If the radius of the inside lane is \( x \) and each lane is 2.5 feet wide, how far apart should the officials start the runners in the inside lane and the outside (6th) lane? (*Hint:* Circumference = \( 2\pi r \), where \( r \) is the radius of the circle).

**NUMBER THEORY** For Exercises 59 and 60, let \( x \) be an odd integer.

59. Write an expression for the next odd integer.
60. Find the product of \( x \) and the next odd integer.

61. **OPEN ENDED** Write a monomial and a trinomial involving one variable. Then find their product.

**CHALLENGE** For Exercises 62–64, use the following information.
An even number can be represented by \( 2x \), where \( x \) is any integer.

62. Show that the product of two even integers is always even.
63. Write a representation for an odd integer.
64. Show that the product of an even and an odd integer is always even.

65. **Writing in Math** Use the information about the area of a rectangle on page 390 to explain how the product of a monomial and a polynomial relate to finding the area of a rectangle. Include the product of \( 2x \) and \( x + 3 \) derived algebraically in your answer.
66. A plumber charges $70 for the first thirty minutes of each house call plus $4 for each additional minute that she works. The plumber charges Ke-Min $122 for her time. How much amount of time, in minutes, did the plumber work?
A 43
B 48
C 58
D 64

67. REVIEW What is the slope of this line?
F \( \frac{5}{2} \)
G -2
H \(-\frac{1}{2}\)
J \(\frac{2}{5}\)

Find each sum or difference. (Lesson 7-4)

68. \((4x^2 + 5x) + (-7x^2 + x)\)

70. \((5b - 7ab + 8a) - (5ab - 4a)\)

69. \((3y^2 + 5y - 6) - (7y^2 - 9)\)

71. \((6p^3 + 3p^2 - 7) + (p^3 - 6p^2 - 2p)\)

State whether each expression is a polynomial. If the expression is a polynomial, identify it as a monomial, a binomial, or a trinomial. (Lesson 7-3)

72. \(4x^2 - 10ab + 6\)

73. \(4c + ab - c\)

74. \(\frac{7}{y} + y^2\)

75. \(\frac{y^3}{3}\)

Define a variable, write an inequality, and solve each problem. Then check your solution. (Lesson 6-3)

76. Six increased by ten times a number is less than nine times the number.

77. Nine times a number increased by four is no less than seven decreased by thirteen times the number.

Write an equation of the line that passes through each pair of points. (Lesson 4-4)

78. \((-3, -8), (1, 4)\)

79. \((-4, 5), (2, -7)\)

80. \((3, -1), (-3, 2)\)

Solve each equation. (Lesson 2-5)

81. \(2(x + 3) + 3 = 4x - 5\)

82. \(3(y - 3) - 6 = 9y - 15\)

83. \(2(3a + 6) - 3 = 6a + 12\)

84. BASKETBALL Tremaine scored 54 three-point field goals, 84 two-point field goals, and 106 free throws in 23 games. How many points did he score on average per game? (Lesson 2-4)

85. \((a)(a)\)

86. \(2x(3x^2)\)

87. \(-3y^2(8y^2)\)

88. \(4y(3y) - 4y(6)\)

89. \(-5n(2n^2) - (-5n)(8n) + (-5n)(4)\)

90. \(3p^2(6p^2) - 3p^2(8p) + 3p^2(12)\)

Simplify. (Lesson 7-1)

85. \((a)(a)\)

86. \(2x(3x^2)\)

87. \(-3y^2(8y^2)\)

88. \(4y(3y) - 4y(6)\)

89. \(-5n(2n^2) - (-5n)(8n) + (-5n)(4)\)

90. \(3p^2(6p^2) - 3p^2(8p) + 3p^2(12)\)
### Algebra Lab

#### Multiplying Polynomials

**ACTIVITY 1** Use algebra tiles to find \((x + 2)(x + 5)\).

The rectangle will have a width of \(x + 2\) and a length of \(x + 5\). Use algebra tiles to mark off the dimensions on a product mat. Then complete the rectangle with algebra tiles.

The rectangle consists of 1 blue \(x^2\)-tile, 7 green \(x\)-tiles, and 10 yellow 1-tiles. The area of the rectangle is \(x^2 + 7x + 10\). Therefore, \((x + 2)(x + 5) = x^2 + 7x + 10\).

**ACTIVITY 2** Use algebra tiles to find \((x - 1)(x - 4)\).

**Step 1** The rectangle will have a width of \(x - 1\) and a length of \(x - 4\). Use algebra tiles to mark off the dimensions on a product mat. Then begin to make the rectangle with algebra tiles.

**Step 2** Determine whether to use 4 yellow 1-tiles or 4 red \(-1\)-tiles to complete the rectangle. Remember that the numbers at the top and side give the dimensions of the tile needed. The area of each tile is the product of \(-1\) and \(-1\) or 1. This is represented by a yellow 1-tile. Fill in the space with 4 yellow 1-tiles to complete the rectangle.

The rectangle consists of 1 blue \(x^2\)-tile, 5 red \(-x\)-tiles, and 4 yellow 1-tiles. The area of the rectangle is \(x^2 - 5x + 4\). Therefore, \((x - 1)(x - 4) = x^2 - 5x + 4\).
Use algebra tiles to find \((x - 3)(2x + 1)\).

**Step 1** The rectangle will have a width of \(x - 3\) and a length of \(2x + 1\). Mark off the dimensions on a product mat. Then make the rectangle.

**Step 2** Determine what color \(x\)-tiles and what color 1-tiles to use to complete the rectangle. The area of each \(-x\)-tile is the product of \(x\) and \(-1\). This is represented by a red \(-x\)-tile. The area of the \(-1\)-tile is represented by the product of \(1\) and \(-1\) or \(-1\). This is represented by a red \(-1\)-tile. Complete the rectangle with 3 red \(-x\)-tiles and 3 red \(-1\)-tiles.

**Step 3** Rearrange the tiles to simplify the diagram. Notice that a zero pair is formed by one positive and one negative \(x\)-tile.

There are 2 blue \(x^2\)-tiles, 5 red \(-x\)-tiles, and 3 red \(-1\)-tiles left.

\[(x - 3)(2x + 1) = 2x^2 - 5x - 3\]

**Model**

Use algebra tiles to find each product.

1. \((x + 2)(x + 3)\)  
2. \((x - 1)(x - 3)\)  
3. \((x + 1)(x - 2)\)  
4. \((x + 1)(2x + 1)\)  
5. \((x - 2)(2x - 3)\)  
6. \((x + 3)(2x - 4)\)

**Analyze the Results**

7. You can also use the Distributive Property to find the product of two binomials. The figure at the right shows the model for \((x + 3)(x + 4)\) separated into four parts. Write a sentence or two explaining how this model shows the use of the Distributive Property.
To compute $24 \times 36$, we multiply each digit in 24 by each digit in 36, paying close attention to the place value of each digit.

<table>
<thead>
<tr>
<th>Step 1</th>
<th>Step 2</th>
<th>Step 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Multiply by the ones.</td>
<td>Multiply by the tens.</td>
<td>Add like place values.</td>
</tr>
<tr>
<td>$24 \times 36$</td>
<td>$24 \times 36$</td>
<td>$24$</td>
</tr>
<tr>
<td>$144$</td>
<td>$144$</td>
<td>$+ 720$</td>
</tr>
<tr>
<td>$720$</td>
<td></td>
<td>$864$</td>
</tr>
</tbody>
</table>

You can multiply two binomials in a similar way.

**EXAMPLE**

### The Distributive Property

Find $(x + 3)(x + 2)$.

**Method 1**  
Vertical  
Multiply by 2.  
Multiply by $x$.  
Combine like terms.  
$x + 3$  
$(x) x + 2$  
$2x + 6$  
$x^2 + 3x$  
$2(x + 3) = 2x + 6$  
$x(x + 3) = x^2 + 3x$  
$x + 3$  
$(x) x + 2$  
$2x + 6$  
$x^2 + 3x$  
$x^2 + 5x + 6$

**Method 2**  
Horizontal  
$(x + 3)(x + 2) = x(x + 2) + 3(x + 2)$  
$= x^2 + x(2) + 3(x) + 3(2)$  
$= x^2 + 2x + 3x + 6$  
$= x^2 + 5x + 6$

Find each product.  
1A. $(m + 4)(m + 5)$  
1B. $(y - 2)(y + 8)$
There is a shortcut version of the Distributive Property called the **FOIL method**. You can use the FOIL method to multiply two binomials.

### Example

**FOIL Method**

Find each product.

**a.** $(x - 5)(x + 7)$

\[
(x - 5)(x + 7) = (x)(x) + (x)(7) + (-5)(x) + (-5)(7) \\
= x^2 + 7x - 5x - 35 \\
= x^2 + 2x - 35
\]

**b.** $(2y + 3)(6y - 7)$

\[
(2y + 3)(6y - 7) = (2y)(6y) + (2y)(-7) + (3)(6y) + (3)(-7) \\
= 12y^2 - 14y + 18y - 21 \\
= 12y^2 + 4y - 21
\]

The FOIL method can be used to find an expression that represents the area of geometric shapes when the lengths of the sides are given as binomials.
EXAMPLE  FOIL Method

3. GEOMETRY The area $A$ of a trapezoid is one half the height $h$ times the sum of the bases, $b_1$ and $b_2$. Write an expression for the area of the trapezoid.

Explore Identify the height and bases.

\[
\begin{align*}
h &= x + 2 \\
b_1 &= 3x - 7 \\
b_2 &= 2x + 1
\end{align*}
\]

Plan Now write and apply the formula.

\[
A = \frac{1}{2} h (b_1 + b_2)
\]

Solve

\[
\begin{align*}
A &= \frac{1}{2} (x + 2) [(3x - 7) + (2x + 1)] \\
&= \frac{1}{2} (x + 2) (5x - 6) \\
&= \frac{1}{2} [x(5x) + x(-6) + 2(5x) + 2(-6)] \\
&= \frac{1}{2} (5x^2 - 6x + 10x - 12) \\
&= \frac{1}{2} (5x^2 + 4x - 12) \\
&= \frac{5}{2} x^2 + 2x - 6
\end{align*}
\]

Check The area of the trapezoid is $\frac{5}{2} x^2 + 2x - 6$ square units.

3. Write an expression for the area of a triangle with a base of $2x + 3$ and a height of $3x - 1$.

Multiply Polynomials The Distributive Property can be used to multiply any two polynomials.

EXAMPLE  The Distributive Property

4. Find each product.

a. $(4x + 9)(2x^2 - 5x + 3)$

\[
\begin{align*}
(4x + 9)(2x^2 - 5x + 3) &= 4x(2x^2 - 5x + 3) + 9(2x^2 - 5x + 3) \\
&= 8x^3 - 20x^2 + 12x + 18x^2 - 45x + 27 \\
&= 8x^3 - 2x^2 - 33x + 27
\end{align*}
\]
b. \((y^2 - 2y + 5)(6y^2 - 3y + 1)\)

\[
(y^2 - 2y + 5)(6y^2 - 3y + 1) = y^2(6y^2 - 3y + 1) - 2y(6y^2 - 3y + 1) + 5(6y^2 - 3y + 1)
\]

\[
= 6y^4 - 3y^3 + y^2 - 12y^3 + 6y^2 - 2y + 30y^2 - 15y + 5
\]

\[
= 6y^4 - 15y^3 + 37y^2 - 17y + 5
\]

4A. \((3x - 5)(2x^2 + 7x - 8)\)  
4B. \((m^2 + 2m - 3)(4m^2 - 7m + 5)\)

**Examples 1–2** (pp. 398–399)

Find each product.

1. \((y + 4)(y + 3)\)  
2. \((x - 2)(x + 6)\)  
3. \((a - 8)(a + 5)\)  
4. \((4h + 5)(h + 7)\)  
5. \((9p - 1)(3p - 2)\)  
6. \((2g + 7)(5g - 8)\)

**Example 3** (p. 400)

GEOMETRY The area \(A\) of a triangle is half the product of the base \(b\) times the height \(h\). Write a polynomial expression that represents the area of the triangle at the right.

**Example 4** (p. 400)

Find each product.

7. \((3k - 5)(2k^2 + 4k - 3)\)  
8. \((k + 4)(y^2 + 2y - 6)\)  
9. \((4x^2 - 2)(2x^2 + 6x + 1)\)  
10. \((k - 3)(2k^2 + 4k - 3)\)  

**Homework**

**For Exercises**  
12–29  
30–33  
34–41  

**See Examples**

1, 2  
3  
4

**Exercises**

Find each product.

12. \((b + 8)(b + 2)\)  
13. \((n + 6)(n + 7)\)  
14. \((x - 4)(x - 9)\)  
15. \((a - 3)(a - 5)\)  
16. \((y + 4)(y - 8)\)  
17. \((p + 2)(p - 10)\)  
18. \((2w - 5)(w + 7)\)  
19. \((k + 12)(3k - 2)\)  
20. \((8d + 3)(5d + 2)\)  
21. \((4k + 3)(9k + 6)\)  
22. \((7x - 4)(5x - 1)\)  
23. \((6a - 5)(3a - 8)\)  
24. \((2n + 3)(2n + 3)\)  
25. \((5m - 6)(5m - 6)\)  
26. \((10r - 4)(10r + 4)\)  
27. \((7t + 5)(7t - 5)\)  
28. \((8x + 2y)(5x - 4y)\)  
29. \((11a - 6b)(2a + 3b)\)

**GEOMETRY** Write an expression to represent the area of each figure.

30. \(2x - 5\)  
31. \(3x - 2\)

32. \(2x - 1\)  
33. \(3x + 4\)

Lesson 7-6 Multiplying Polynomials 401
Find each product.
34. \((p + 4)(p^2 + 2p - 7)\)
35. \((a - 3)(a^2 - 8a + 5)\)
36. \((2x - 5)(3x^2 - 4x + 1)\)
37. \((3k + 4)(7k^2 + 2k - 9)\)
38. \((n^2 - 3n + 2)(n^2 + 5n - 4)\)
39. \((y^2 + 7y - 1)(y^2 - 6y + 5)\)

Simplify.
40. \((m + 2)[(m^2 + 3m - 6) + (m^2 - 2m + 4)]\)
41. \([(t^2 + 3t - 8) - (t^2 - 2t + 6)](t - 4)\)

**GEOMETRY** The volume \(V\) of a prism equals the area of the base \(B\) times the height \(h\). Write an expression to represent the volume of each prism.

42. \[\begin{array}{c}
a + 1 \\
2a - 2 \\
a + 5
\end{array}\]
43. \[\begin{array}{c}
3y \\
2y \\
6
\end{array}\]

44. **BASKETBALL** The dimensions of a professional basketball court are represented by a width of \(5y - 6\) feet and a length of \(2y + 10\) feet. Find an expression for the area of the court.

**OFFICE SPACE** For Exercises 45–47, use the following information.
LaTanya’s modular office is square. Her office in the company’s new building will be 2 feet shorter in one direction and 4 feet longer in the other.

45. Write expressions for the dimensions of LaTanya’s new office.
46. Write a polynomial expression for the area of her new office.
47. Suppose her office is presently 9 feet by 9 feet. Will her new office be bigger or smaller than her old office and by how much? Explain.

48. **POOL CONSTRUCTION** A homeowner is installing a swimming pool in his backyard. He wants its length to be 4 feet longer than its width. Then he wants to surround it with a concrete walkway 3 feet wide. If he can only afford 300 square feet of concrete for the walkway, what should the dimensions of the pool be?

49. **REASONING** Compare and contrast the procedure used to multiply a trinomial by a binomial using the vertical method with the procedure used to multiply a three-digit number by a two-digit number.

50. **ALGEBRA TILES** Draw a diagram to show how you would use algebra tiles to find the product of \(2x - 1\) and \(x + 3\).

51. **CHALLENGE** Determine whether the following statement is *sometimes*, *always*, or *never* true. Explain your reasoning.
   *The product of a binomial and a trinomial is a polynomial with four terms.*
52. OPEN ENDED Write a binomial and a trinomial involving a single variable. Then find their product.

53. **Writing in Math** Using the information about multiplying binomials on page 398 explain how multiplying binomials is similar to multiplying two-digit numbers. Include a demonstration of a horizontal method for multiplying $24 \times 36$ in your answer.

54. A rectangle’s width is represented by $x$ and its length by $y$. Which expression best represents the area of the rectangle if the length and width are doubled?

- A $2xy$
- B $2(xy)^2$
- C $4xy$
- D $(xy)^2$

55. REVIEW Tania’s age is 4 years less than twice her little brother Billy’s age. If Tania is 12, which equation can be used to determine Billy’s age?

- F $x = 12$
- G $12 = 4 - 2x$
- H $12(2) - 4 = x$
- J $12 = 2x - 4$

---

**Simplify.** (Lesson 7-5)

56. $3x(2x - 4) + 6(5x^2 + 2x - 7)$

57. $4a(5a^2 + 2a - 7) - 3(2a^2 - 6a - 9)$

**GEOMETRY** The sum of the degree measures of the angles of a triangle is 180. (Lesson 7-4)

a. Write an expression to represent the measure of the third angle of the triangle.

b. If $x = 15$, find the measures of the three angles of the triangle.

If $f(x) = 2x - 5$ and $g(x) = x^2 + 3x$, find each value. (Lesson 3-6)

59. $f(-4)$

60. $g(-2) + 7$

61. $f(a + 3)$

Solve each equation or formula for the variable specified. (Lesson 2-8)

62. $a = \frac{v}{t}$ for $t$

63. $ax - by = 2cz$ for $y$

64. $4x + 3y = 7$ for $y$

Solve each equation. (Lesson 2-4)

65. $\frac{d - 2}{3} = 7$

66. $3n + 6 = -15$

67. $35 + 20t = 100$

---

**PREREQUISITE SKILL** Simplify. (Lesson 7-1)

68. $(6a)^2$

69. $(7x)^2$

70. $(9b)^2$

71. $(4y^2)^2$

72. $(2v^3)^2$

73. $(3g^4)^2$
Special Products

In the previous lesson, you learned how to multiply two binomials using the FOIL method. You may have noticed that the Outer and Inner terms often combine to produce a trinomial product. This is not always the case, however. Notice that the product of \(x + 3\) and \(x - 3\) is a binomial.

\[
(x + 5)(x - 3) \quad \text{FOIL} \quad = x^2 - 3x + 5x - 15 \\
= x^2 + 2x - 15
\]

\[
(x + 3)(x - 3) \quad \text{FOIL} \quad = x^2 - 3x + 3x - 9 \\
= x^2 + 0x - 9 \\
= x^2 - 9
\]

**Squares of Sums and Differences** While you can always use the FOIL method to find the product of two binomials, some pairs of binomials have products that follow a specific pattern. One such pattern is the square of a sum, \((a + b)^2\) or \((a + b)(a + b)\).

\[
(a + b)^2 = a^2 + 2ab + b^2
\]

**KEY CONCEPT**

**Square of a Sum**

**Words** The square of \(a + b\) is the square of \(a\) plus twice the product of \(a\) and \(b\) plus the square of \(b\).

**Symbols** \((a + b)^2 = (a + b)(a + b) = a^2 + 2ab + b^2\)

**Example** \((x + 7)^2 = x^2 + 2(x)(7) + 7^2 = x^2 + 14x + 49\)

**EXAMPLE** Square of a Sum

Find \((4y + 5)^2\).

\[
(a + b)^2 = a^2 + 2ab + b^2
\]

\[
(4y + 5)^2 = (4y)^2 + 2(4y)(5) + 5^2 \quad \text{for } a = 4y \text{ and } b = 5
\]

\[
= 16y^2 + 40y + 25 \quad \text{Check by using FOIL.}
\]
Find each product.

1A. \((8c + 3d)^2\)

1B. \((3x + 4y)^2\)

To find the pattern for the square of a difference, \((a - b)^2\), write \(a - b\) as \(a + (-b)\) and square it using the square of a sum pattern.

\[
(a - b)^2 = [a + (-b)]^2
= a^2 + 2(a)(-b) + (-b)^2
= a^2 - 2ab + b^2
\]

Square of a Sum
Simplify. Note that \((-b)^2 = (-b)(-b)\) or \(b^2\).

\[
(a - b)^2 = a^2 - 2ab + b^2
\]

**EXAMPLE**

**Square of a Difference**

Find \((5m^3 - 2n)^2\).

\[
(a - b)^2 = a^2 - 2ab + b^2
\]

\[
(5m^3 - 2n)^2 = (5m^3)^2 - 2(5m^3)(2n) + (2n)^2
= 25m^6 - 20m^3n + 4n^2
\]

Simplify.

**CHECK Your Progress**

Find each product.

2A. \((6p - 1)^2\)

2B. \((a - 2b)^2\)

**GENETICS** The Punnett square shows the possible gene combinations between two hamsters. Each hamster passes on one dominant gene \(G\) for golden coloring and one recessive gene \(g\) for cinnamon coloring.

Show how combinations can be modeled by the square of a binomial. Then determine what percent of the offspring will be pure golden, hybrid golden, and pure cinnamon.

Each parent has half the genes necessary for golden coloring and half the genes necessary for cinnamon coloring. The makeup of each parent can be modeled by \(0.5G + 0.5g\). Their offspring can be modeled by the product of \(0.5G + 0.5g\) and \(0.5G + 0.5g\) or \((0.5G + 0.5g)^2\).
Use this product to determine possible colors of the offspring.

\[(a + b)^2 = a^2 + 2ab + b^2\]  \hspace{1cm} \text{Square of a Sum}

\[(0.5G + 0.5g)^2 = (0.5G)^2 + 2(0.5G)(0.5g) + (0.5g)^2\]
\[= 0.25G^2 + 0.5Gg + 0.25g^2\]
\[= 0.25GG + 0.5Gg + 0.25gg\]

Thus, 25% of the offspring are GG or pure golden, 50% are Gg or hybrid golden, and 25% are gg or pure cinnamon.

3. Andrew has a garden that is \(x\) feet long and \(x\) feet wide. He decides that he wants to add 3 feet to the length and the width in order to grow more vegetables. Show how the new area of the garden can be modeled by the square of a binomial.

**Product of a Sum and a Difference** You can use the diagram below to find the pattern for the product of the sum and difference of the same two terms, \((a + b)(a - b)\). Recall that \(a - b\) can be rewritten as \(a + (-b)\).

The resulting product, \(a^2 - b^2\), is called the **difference of two squares**.

<table>
<thead>
<tr>
<th>Key Concept</th>
<th>Product of a Sum and a Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Words</strong></td>
<td>The product of (a + b) and (a - b) is the square of (a) minus the square of (b).</td>
</tr>
<tr>
<td><strong>Symbols</strong></td>
<td>((a + b)(a - b) = (a - b)(a + b) = a^2 - b^2)</td>
</tr>
<tr>
<td><strong>Example</strong></td>
<td>((x + 9)(x - 9) = x^2 - 9^2 = x^2 - 81)</td>
</tr>
</tbody>
</table>

**EXAMPLE**

Find \((11v - 8w^2)(11v + 8w^2)\)

\[(a - b)(a + b) = a^2 - b^2\]
\[(11v - 8w^2)(11v + 8w^2) = (11v)^2 - (8w^2)^2\]
\[= 121v^2 - 64w^4\]  \hspace{1cm} \text{Simplify.}

Find each product.

4A. \((3n + 2)(3n - 2)\)  \hspace{1cm} 4B. \((4c - 7d)(4c + 7d)\)
Find each product.
1. \((a + 6)^2\)
2. \((2a + 7b)^2\)
3. \((3x + 9y)^2\)
4. \((4n - 3)(4n - 3)\)
5. \((x^2 - 6y)^2\)
6. \((9 - p)^2\)

**GENETICS** For Exercises 7 and 8, use the following information.
Dalila has brown eyes and Bob has blue eyes. Brown genes \(B\) are dominant over blue genes \(b\). A person with genes \(BB\) or \(Bb\) has brown eyes. Someone with genes \(bb\) has blue eyes. Suppose Dalila’s genes for eye color are \(Bb\).
7. Write an expression for the possible eye coloring of Dalila and Bob’s children.
8. What is the probability that a child of Dalila and Bob would have blue eyes?

**Example 4** (p. 406)
Find each product.
9. \((8x - 5)(8x + 5)\)
10. \((3a + 7b)(3a - 7b)\)
11. \((4y^2 + 3z)(4y^2 - 3z)\)

**Cystic Fibrosis**
Cystic fibrosis is inherited from parents only if both parents have the abnormal \(CF\) gene. Children of two parents with the \(CF\) gene will either be affected with the disease, a carrier but not affected, or not have the gene.

18. Write an expression for the genetic makeup of children of two parents that are carriers of cystic fibrosis.
19. What is the probability that a child will not be affected and not be a carrier?

Find each product.
20. \((b + 7)(b - 7)\)
21. \((c - 2)(c + 2)\)
22. \((11r + 8)(11r - 8)\)

Find each product.
23. \((9x + 3)^2\)
24. \((4 - 6h)^2\)
25. \((12p - 3)(12p + 3)\)
26. \((a + 5b)^2\)
27. \((m + 7n)^2\)
28. \((2x - 9y)^2\)
29. \((3n - 10p)^2\)
30. \((5w + 14)(5w - 14)\)
31. \((4d - 13)(4d + 13)\)
32. \((x^3 + 4y)^2\)
33. \((3a^2 - b^2)^2\)
34. \((8a^2 - 9b^3)(8a^2 + 9b^3)\)
35. \((5x^4 - y)(5x^4 + y)\)
36. \(\left(\frac{2}{3}x - 6\right)^2\)
37. \(\left(\frac{4}{5}x + 10\right)^2\)
38. \((2n + 1)(2n - 1)(n + 5)\)
39. \((p + 3)(p - 4)(p - 3)(p + 4)\)
**MAGIC TRICK** For Exercises 40–43, use the following information. Madison says that she can perform a magic trick with numbers. She asks you to pick an integer, any integer. Square that integer. Then, add twice your original number. Next add 1. Take the square root of the result. Finally, subtract your original number. Then Madison exclaims with authority, “Your answer is 1!”

40. Pick an integer and follow Madison’s directions. Is your result 1?
41. Let \( a \) represent the integer you chose. Then, find a polynomial representation for the first three steps of Madison’s directions.
42. The polynomial you wrote in Exercise 41 is the square of what binomial sum?
43. Take the square root of the perfect square you wrote in Exercise 42, then subtract \( a \), your original integer. What is the result?

**WRESTLING** For Exercises 44–46, use the following information.
A high school wrestling mat must be a square with 38-foot sides and contain two circles as shown. Suppose the inner circle has a radius of \( s \) feet, and the outer circle’s radius is nine feet longer than the inner circle.

44. Write an expression for the area of the larger circle.
45. Write an expression for the area of the square outside the circle.
46. Use the expression to find the area if \( s = 1 \).

47. **GEOMETRY** The area of the shaded region models the difference of two squares, \( a^2 - b^2 \). Show that the area of the shaded region is also equal to \((a - b)(a + b)\). (Hint: Divide the shaded region into two trapezoids as shown.)

48. **REASONING** Compare and contrast the pattern for the square of a sum with the pattern for the square of a difference.

49. **ALGEBRA TILES** Draw a diagram to show how you would use algebra tiles to model the product of \( x - 3 \) and \( x - 3 \), or \((x - 3)^2\).

50. **OPEN ENDED** Write two binomials whose product is a difference of squares. Then multiply to verify your answer.

51. **CHALLENGE** Does a pattern exist for the cube of a sum, \((a + b)^3\)?
   a. Investigate this question by finding the product of \((a + b)(a + b)(a + b)\).
   b. Use the pattern you discovered in part a to find \((x + 2)^3\).
   c. Draw a diagram of a geometric model for the cube of a sum.

52. **Writing in Math** Using the information about the product of two binomials on page 404 distinguish when the product of two binomials is also a binomial. Include an example of two binomials whose product is a binomial and an example of two binomials whose product is not a binomial in your answer.

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**Real-World Link**
Cael Sanderson of Iowa State University is the only wrestler in NCAA Division 1 history to be undefeated for four years. He compiled a 159–0 record from 1999–2002.

Source: www.teamsanderson.cc
53. The base of a triangle is represented by \( x - 4 \), and the height is represented by \( x + 4 \). Which of the following represents the area of the triangle?
   - A. \( x^2 - 16 \)
   - B. \( \frac{1}{2}x^2 + 4x - 8 \)
   - C. \( x^2 + 8x - 16 \)
   - D. \( \frac{1}{2}x^2 - 8 \)

54. **REVIEW** The sum of a number and 8 is –19. Which equation shows this relationship?
   - F. \( 8n = -19 \)
   - G. \( n + 8 = -19 \)
   - H. \( n - 8 = 19 \)
   - J. \( n - 19 = 8 \)

---

**Spiral Review**

Find each product.  (Lesson 7-6)

55. \((x + 2)(x + 7)\)
56. \((c - 9)(c + 3)\)
58. \((3n - 5)(8n + 5)\)
59. \((x - 2)(3x^2 - 5x + 4)\)
60. \((2k + 5)(2k^2 - 8k + 7)\)

Solve.  (Lesson 7-5)

61. \(6(x + 2) + 4 = 5(3x - 4)\)
62. \(-3(3a - 8) + 2a = 4(2a + 1)\)
63. \(p(p + 2) + 3p = p(p - 3)\)
64. \(y(y - 4) + 2y = y(y + 12) - 7\)

Use elimination to solve each system of equations.  (Lesson 5-3, 5-4)

65. \(\frac{3}{4}x + \frac{1}{5}y = 5\)
   \(\frac{3}{4}x - \frac{1}{5}y = -5\)
66. \(2x - y = 10\)
   \(5x + 3y = 3\)
67. \(2x = 4 - 3y\)
   \(3y - x = -11\)

Write the slope-intercept form of an equation that passes through the given point and is perpendicular to the graph of each equation.  (Lesson 4-6)

68. \(5x + 5y = 35, (-3, 2)\)
69. \(2x - 5y = 3, (-2, 7)\)
70. \(5x + y = 2, (0, 6)\)

Find the \(n\)th term of each arithmetic sequence described.  (Lesson 3-7)

71. \(a_1 = 3, d = 4, n = 18\)
72. \(-5, 1, 7, 13, \ldots \) for \(n = 12\)

73. **PHYSICAL FITNESS** Mitchell likes to exercise regularly. He likes to warm up by walking two miles. Then he runs five miles. Finally, he cools down by walking another mile. Identify the graph that best represents Mitchell’s heart rate as a function of time.  (Lesson 1-9)
Key Concepts

Multiplying Monomials (Lesson 7-1)
- To multiply two powers that have the same base, add exponents.
- To find the power of a power, multiply exponents.
- The power of a product is the product of the powers.

Dividing Monomials (Lesson 7-2)
- To divide two powers that have the same base, subtract the exponents.
- To find the power of a quotient, find the power of the numerator and the power of the denominator.
- Any nonzero number raised to the zero power is 1.
- For any nonzero number \(a\) and any integer \(n\),
  \[a^{-n} = \frac{1}{a^n}\] and \[\frac{1}{a^{-n}} = a^n\].

Polynomials (Lesson 7-3)
- The degree of a monomial is the sum of the exponents of all its variables.
- The degree of a polynomial is the greatest degree of any term. To find the degree of a polynomial, you must find the degree of each term.

Operations with Polynomials (Lessons 7-4 to 7-7)
- The Distributive Property can be used to multiply a polynomial by a monomial.
- Square of a Sum: \((a + b)^2 = a^2 + 2ab + b^2\)
- Square of a Difference: \((a - b)^2 = a^2 - 2ab + b^2\)
- Product of a Sum and a Difference: \((a + b)(a - b) = (a - b)(a + b) = a^2 - b^2\)

Vocabulary Check
Choose a term from the vocabulary list that best matches each example.

1. \(4^{-3} = \frac{1}{4^3}\)
2. \((n^3)^5 = n^{15}\)
3. \(\frac{4x^2y}{8xy^3} = \frac{x}{2y^2}\)
4. \(4x^2\)
5. \(x^2 - 3x + 1\)
6. \(2^0 = 1\)
7. \(x^4 - 3x^3 + 3x^2 - 1\)
8. \(x^2 + 2\)
9. \((a^3b)(2ab^2) = 2a^4b^3\)
10. \((x + 3)(x - 4) = x^2 - 4x + 3x - 12\)
Lesson-by-Lesson Review

7–1 Multiplying Monomials (pp. 358–364)

Simplify.
11. \(y^3 \cdot y^3 \cdot y\)
12. \((3ab)(-4a^2b^3)\)
13. \((-4a^2x)(-5a^2x^4)\)
14. \((4a^2b)^3\)
15. \((-3xy)^2(4x)^3\)
16. \((-2c^2d)^4(-3c^2)^3\)
17. \(-\frac{1}{2} (m^2n^4)^2\)
18. \((5a^2)^3 + 7(a^6)\)

19. GEOMETRY A cone has a radius of \(4x^3\) and a height of \(3b^2\). Use the formula \(V = \frac{1}{3} \pi r^2h\) to find the volume of the cone.

7–2 Dividing Monomials (pp. 366–373)

Simplify. Assume that no denominator is equal to zero.
20. \(\frac{(3y)^0}{6a}\)
21. \(\left(\frac{3bc^2}{4d}\right)^3\)
22. \(x^{-2}y^{0}z^3\)
23. \(\frac{27b-2}{14b-3}\)
24. \(\frac{(3a^3bc^2)^2}{18a^2b^3c^4}\)
25. \(-\frac{16a^3b^2x^4y}{-48a^6bxy^3}\)
26. \(\frac{(-a)^5b^{08}}{a^5b^2}\)
27. \(\frac{(4a^{-1})^{-2}}{(2a)^2}\)
28. \(\frac{5xy^{-2}}{35x^{-2}y^6}\)
29. \(\frac{12}{3} \left(\frac{m}{n}\right)^3 \left(\frac{n}{m}\right)^4\)

30. GEOMETRY The area of a triangle is 50a²b square feet. The base of the triangle is 5a feet. What is the height of the triangle?

\[
\text{Area} = \frac{1}{2} \times \text{base} \times \text{height}
\]

\[
50a^2b = \frac{1}{2} \times 5a \times \text{height}
\]

\[
\text{height} = \frac{50a^2b}{\frac{1}{2} \times 5a} = \frac{50a^2b}{2.5a} = 20ab
\]

Example 1 Simplify \((2ab^2)(3a^2b^3)\).

\((2ab^2)(3a^2b^3) = (2 \cdot 3)(a \cdot a^2)(b^2 \cdot b^3)\)

\(= 6a^3b^5\)

Example 2 Simplify \((2x^2y^3)^3\).

\((2x^2y^3)^3 = 2^3(x^2)^3(y^3)^3\)

\(= 8x^6y^9\)

Example 3 Simplify \(\frac{2x^6y}{8x^2y^2}\). Assume that no denominator is equal to zero.

\[
\frac{2x^6y}{8x^2y^2} = \left(\frac{2}{8}\right) \left(\frac{x^6}{x^2}\right) \left(\frac{y}{y^2}\right)
\]

\(= \frac{1}{4} \left(x^{6-2}\right) \left(y^{1-2}\right)\)

\(= \frac{x^4}{4y}\)

Example 4 Simplify \(\frac{m^{-4}n^3p^0}{mn^{-2}}\). Assume that no denominator is zero.

\[
\frac{m^{-4}n^3p^0}{mn^{-2}} = \left(\frac{m^{-4}}{m}\right) \left(\frac{n^3}{n^{-2}}\right) (p^0)
\]

\(= \left(m^{-4-1}\right) \left(n^{3+2}\right)\)

\(= \frac{n^5}{m^5}\)

Simplify.
Polynomials (pp. 376–381)

Find the degree of each polynomial.
31. \( n - 2p^2 \)
32. \( 29n^2 + 17n^2t^2 \)
33. \( 4xy + 9x^3z^2 + 17rs^3 \)
34. \( -6x^3y - 2y^4 + 4 - 8y^2 \)

Arrange the terms of each polynomial so that the powers of \( x \) are in descending order.
35. \( 3x^4 - x + x^2 - 5 \)
36. \( -2x^2y^3 - 27 - 4x^4 + xy + 5x^3y^2 \)

Example 5 Find the degree of \( 2xy^3 + x^2y \).
Polynomial: \( 2xy^3 + x^2y \)
Terms: \( 2xy^3 \), \( x^2y \)
Degree of Each Term: 4, 3
Degree of Polynomial: 4

Example 6 Arrange the terms of \( 4x^2 + 9x^3 - 2 - x \) so that the powers of \( x \) are in descending order.
\( 4x^2 + 9x^3 - 2 - x = 4x^2 + 9x^3 - x^0 - x^1 \) \( x^0 = 1 \) and \( x = x^1 \)
\( = 9x^3 + 4x^2 - x - 2 \) \( 3 > 2 > 1 > 0 \)

CONSTRUCTION Ben is building a brick patio with pavers using the drawing below. Write a polynomial to represent the area of the patio.

Adding and Subtracting Polynomials (pp. 384–388)

Find each sum or difference.
38. \( (2x^2 - 5x + 7) - (3x^3 + x^2 + 2) \)
39. \( (x^2 - 6xy + 7y^2) + (3x^2 + xy - y^2) \)
40. \( (7z^2 + 4) - (3z^2 + 2z - 6) \)
41. \( (13m^4 - 7m - 10) + (8m^4 - 3m + 9) \)
42. \( (11m^2n^2 + 4mn - 6) + (5m^2n^2 + 6mn + 17) \)
43. \( (-5p^2 + 3p + 49) - (2p^2 + 5p + 24) \)

Example 7 Find \( (7r^2 + 9r) - (12r^2 - 4) \).
\( (7r^2 + 9r) - (12r^2 - 4) = 7r^2 + 9r + (-12r^2 + 4) \) The additive inverse of \( 12r^2 - 4 \) is \(-12r^2 + 4\).
\( = (7r^2 - 12r^2) + 9r + 4 \) Group like terms.
\( = -5r^2 + 9r + 4 \) Add like terms.

Example 8 Find \( (14m^3 - 2x + 5) + (x^2 - 6xy + 7y^2) \).
\( (14m^3 - 2x + 5) + (x^2 - 6xy + 7y^2) = 14m^3 - 2x + 5 + x^2 - 6xy + 7y^2 \)
\( = 14m^3 + x^2 - 6xy + 5 + 7y^2 - 2x \) Combine like terms.
Multiplying a Polynomial by a Monomial (pp. 390–395)

Simplify.
45. \(b(4b - 1) + 10b\)
46. \(x(3x - 5) + 7(x^2 - 2x + 9)\)
47. \(8y(11y^2 - 2y + 13) - 9(3y^3 - 7y + 2)\)
48. \(2x(x - y^2 + 5) - 5y^2(3x - 2)\)

Solve each equation.
49. \(m(2m - 5) + m = 2m(m - 6) + 16\)
50. \(2(3w + w^2) - 6 = 2w(w - 4) + 10\)

51. **SHOPPING** Nicole bought \(x\) shirts for $15.00 each, \(y\) pants for $25.72 each, and \(z\) belts for $12.53 each. Sales tax on these items was 7%. Write an expression to find the total cost of Nicole’s purchases.

Example 8 Simplify \(x^2(x + 2) + 3(x^3 + 4x^2)\).
\[
x^2(x + 2) + 3(x^3 + 4x^2) = x^2(x) + x^2(2) + 3(x^3 + 3(4x^2))
\]
\[
= x^3 + 2x^2 + 3x^3 + 12x^2 \\
= 4x^3 + 14x^2 \\
\text{Combine like terms.}
\]

Example 9 Solve
\[
x(x - 10) + x(x + 2) + 3 = 2x(x + 1) - 7.
\]
\[
x(x - 10) + x(x + 2) + 3 = 2x^2 + 2x - 7
\]
\[
2x^2 - 8x + 3 = 2x^2 + 2x - 7
\]
\[
-8x + 3 = 2x - 7
\]
\[
-10x = -10
\]
\[
x = 1
\]

Multiplying Polynomials (pp. 398–403)

Find each product.
52. \((r - 3)(r + 7)\)
53. \((4a - 3)(a + 4)\)
54. \((5r - 7s)(4r + 3s)\)
55. \((3x + 0.25)(6x - 0.5)\)
56. \((2k + 1)(k^2 + 7k - 9)\)
57. \((4p - 3)(3p^2 - p + 2)\)

58. **MANUFACTURING** A company is designing a box in the shape of a rectangular prism for dry pasta. The length is 2 inches more than twice the width and the height is 3 inches more than the length. Write an expression for the volume of the box.

Example 10 Find \((3x + 2)(x - 2)\).
\[
(3x + 2)(x - 2) = (3x)(x) + (3x)(-2) + (2)(x) + (2)(-2)
\]
\[
= 3x^2 - 6x + 2x - 4 \\
= 3x^2 - 4x - 4 \\
\text{Combine like terms.}
\]

Example 11 Find \((2y - 5)(4y^2 + 3y - 7)\).
\[
(2y - 5)(4y^2 + 3y - 7) = 2y(4y^2 + 3y - 7) - 5(4y^2 + 3y - 7)
\]
\[
= 8y^3 + 6y^2 - 14y - 20y^2 - 15y + 35
\]
\[
= 8y^3 - 14y^2 - 29y + 35
\]
Study Guide and Review

7–7 Special Products (pp. 404–409)

Find each product.

59. \((x - 6)(x + 6)\)
60. \((4x + 7)^2\)
61. \((8x - 5)^2\)
62. \((5x - 3y)(5x + 3y)\)
63. \((6a - 5b)^2\)
64. \((3m + 4n)^2\)

65. **GENETICS** Emily and Santos are both able to roll their tongues. Tongue rolling genes \(R\) are dominant over nonrolling genes \(r\). A person with genes \(RR\) or \(Rr\) is able to roll their tongue. Someone with genes \(rr\) cannot roll their tongue. Suppose Emily’s and Santos’s genes for tongue rolling are \(Rr\). Write an expression for the possible tongue-rolling abilities of Santos’s and Emily’s children. What is the probability that a child of Emily and Santos could not roll their tongue?

**Example 12** Find \((r - 5)^2\).

\[
(a - b)^2 = a^2 - 2ab + b^2 \quad \text{Square of a Difference}
\]

\[
(r - 5)^2 = r^2 - 2(r)(5) + 5^2 \quad a = r \quad b = 5
\]

\[
= r^2 - 10r + 25 \quad \text{Simplify.}
\]

**Example 13** Find \((2c + 9)(2c - 9)\).

\[
(a + b)(a - b) = a^2 - b^2
\]

\[
(2c + 9)(2c - 9) = (2c)^2 - 9^2 \quad a = 2c \quad b = 9
\]

\[
= 4c^2 - 81 \quad \text{Simplify.}
\]
Simplify. Assume that no denominator is equal to zero.

1. \((a^2b^4)(a^3b^5)\)
2. \((-12abc)(4a^2b^4)\)
3. \(\left(\frac{3}{5}m\right)^2\)
4. \((-3a)^4(a^5b)^2\)
5. \((-5a^2)(-6b^3)^2\)
6. \(\frac{mn^4}{m^3n^2}\)
7. \(\frac{9a^2bc^2}{63a^4bc}\)
8. \(\frac{48a^2bc^5}{(3ab^3c^2)^2}\)

Find the degree of each polynomial. Then arrange the terms so that the powers of \(y\) are in descending order.

9. \(2y^2 + 8y^4 + 9y\)
10. \(5xy - 7 + 2y^4 - x^2y^3\)

Find each sum or difference.

11. \((5a + 3a^2 - 7a^3) + (2a - 8a^2 + 4)\)
12. \((x^3 - 3x^2y + 4xy^2 + y^3) - (7x^3 + x^2y - 9xy^2 + y^3)\)

13. **GEOMETRY** The measures of two sides of a triangle are given. If the perimeter is represented by \(11x^2 - 29x + 10\), find the measure of the third side.

14. **MULTIPLE CHOICE** What is the area of the square with sides that measure \(x - 6\)?
   A. \(4x - 24\)
   B. \(x^2 - 12x + 36\)
   C. \(x^2 + 12x + 36\)
   D. \(x^2 - 36\)

15. \((h - 5)^2\)
16. \((4x - y)(4x + y)\)
17. \(3x^2y^3(2x - xy^2)\)
18. \((2a^2b + b^2)^2\)
19. \((4m + 3n)(2m - 5n)\)
20. \((2c + 5)(3c^2 - 4c + 2)\)

Solve each equation.

21. \(2x(x - 3) = 2(x^2 - 7) + 2\)
22. \(3a(a^2 + 5) - 11 = a(3a^2 + 4)\)

23. **MULTIPLE CHOICE** If \(x^2 + 2xy + y^2 = 8\), find \(3(x + y)^2\).
   F. \(2\)
   H. \(12\)
   G. \(4\)
   J. \(24\)

**GENETICS** The Punnett square shows the possible gene combinations of a cross between two pea plants. Each plant passes on one dominant gene \(T\) for tallness and one recessive gene \(t\) for shortness.

24. Show how the possible combinations can be modeled by the square of a binomial.
25. What is the probability that the offspring will be pure tall (\(TT\)), hybrid tall (\(Tt\)), and pure short (\(tt\))?
Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

1. Which expression describes the area in square units of a rectangle that has a width of \(2a^2b\) and a length of \(4a^3b^4\)?
   - A \(8a^6b^4\)
   - B \(8a^5b^5\)
   - C \(6a^6b^4\)
   - D \(6a^5b^5\)

2. **GRIDDABLE** What is the value of \(r\) in the equation below?
   \[
r = \frac{5(2) - 3}{2(7 - 6)}
   \]

3. Which statement is true for the graph below?

![Cost to Rent a Raft graph]

   - F It will cost Chelsea $200 to rent a raft for 4 hours.
   - G It will cost Maria $150 to rent a raft for 2 hours.
   - H It will cost John $100 to rent a raft for 1 hour.
   - J It will cost Marcus $100 to rent a raft for 3 hours.

4. The graph of the linear equation \(y = \frac{1}{2}x + 3\) is shown below.

![Graph of linear equation]

Which point is not in the solution set of \(y > \frac{1}{2}x + 3\)?
   - A (1, 5)
   - B (–6, 1)
   - C (–3, 4)
   - D (–2, 1)

5. Which point on the grid below satisfies the conditions \(x < 5\) and \(y > -6\)?

![Point grid]

   - F Point A
   - G Point B
   - H Point C
   - J Point D

6. Maya used 18 square feet of material to make a blanket. The blanket is in the shape of a square. She would like to put a binding around the outside of the blanket. How can she find the length of a side?
   - A Find the square root of the area.
   - B Multiply the area by 4.
   - C Divide the area by 4.
   - D Divide the area by 2.
7. Describe the effect on the area of a circle when the radius is tripled.

- F The area is reduced by \( \frac{1}{3} \).
- G The area remains constant.
- H The area is tripled.
- J The area is increased nine times.

8. **GRIDDABLE** Bradley needs to stain his deck. The deck measures 14 by 16 feet. If the stain costs $1.25 per square foot, including tax, how much will it cost to stain his deck?

9. The table shows the sum of the interior angle measures in certain convex polygons.

<table>
<thead>
<tr>
<th>Convex Polygon</th>
<th>Number of Sides</th>
<th>Sum of Angle Measures</th>
</tr>
</thead>
<tbody>
<tr>
<td>triangle</td>
<td>3</td>
<td>180°</td>
</tr>
<tr>
<td>quadrilateral</td>
<td>4</td>
<td>360°</td>
</tr>
<tr>
<td>pentagon</td>
<td>5</td>
<td>540°</td>
</tr>
<tr>
<td>hexagon</td>
<td>6</td>
<td>720°</td>
</tr>
</tbody>
</table>

Based on the table, what is the sum of the angle measures of an octagon?

- A 720°
- B 900°
- C 1000°
- D 1080°

10. Carlos builds circular parachutes. The area of one parachute is 100 square feet. If he triples the radius of the parachute to build a new parachute, what will the area of the new parachute be?

- F 100 ft²
- G 300 ft²
- H 600 ft²
- J 900 ft²

11. The net of a square pyramid is shown below.

Measure the dimensions of the pyramid to the nearest \( \frac{1}{8} \) inch. Find the surface area of the pyramid to the nearest square inch.

- A 3 in²
- B 4 in²
- C 6 in²
- D 8 in²

**Pre-AP**

Record your answers on a sheet of paper. Show your work.

12. Two cars leave at the same time and both drive to Nashville. The cars’ distance from Knoxville, in miles, can be represented by the two equations below, where \( t \) represents time in hours.

Car A: \( A = 65t + 10 \);
Car B: \( B = 55t + 20 \)

**a.** Which car is faster? Explain.

**b.** How far did Car B travel after 2 hours?

**c.** Find an expression that models the distance between the two cars.

**d.** How far apart are the cars after \( 3\frac{1}{2} \) hours?